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Pollution Charges, Waste Assimilative Capacity Investment, and Water Quality: The Public Costs of a Public Good

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State of Ohio
Water Resources Center
Ohio State University

Pollution Charges, Waste Assimilative Capacity Investment, and Water Quality: The Public Costs of a Public Good

by

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Preface

This Final Project Report consists of two essentially independent papers. The first, which makes up the body of the report, examines the relationship between pollution charges income and the costs of waste assimilative capacity augmentation, where both have been set so as to maximize the net benefits associated with a river. An earlier version of this paper was presented on November 12, 1968, to the Microanalysis and Econometrics Workshop of the Ohio State University, Department of Economics.

The second part, presented here as an Appendix, derives parametric cost functions for several waste treatment technologies. These functions are specified so that marginal cost functions, important to economic optimization, may be obtained.

I wish to thank Richard Tybout and the other members of the Micro Workshop for their comments. Marilyn Eisenberg has aided in checking the algebra and proofreading, and William Brueggeman and William Gasser have done yeoman service in searching the literature and offering ready forums for discussion. The responsibility for remaining errors must, of course, rest with the author.

I. Introduction

At least since the time of Pigou (1a), economists have advocated taxing or charging activities such as water pollution or air pollution, with the aim of improving the efficiency of the market mechanism in these areas. Frank Knight (1b) early recognized that Pigou's tax was formally identical to the results of the operation of a competitive, private property economy. The implications of this insight have been explored by Mohring and Boyd (1c). In the case of water pollution, replication of the market mechanism would indeed involve a charge for pollutants discharged, which would induce polluters to recognize in their behavior the marginal opportunity costs of utilizing rivers or other bodies of water in this fashion. This opportunity cost is the value of downstream water quality forgone (at the margin) due to the pollutant discharges.

Water quality has the essential joint-supply properties of a public consumption good. Strangely, its "production" by charging for upstream pollution is done at negative cost to the allocating authority, since the income from these charges accrues as a pure rent to, for example, the river. In addition, it may be desired to levy charges on water quality enjoyers for equity reasons, further increasing the portion of the river-rent accruing to the river authority.[1d]

[1a] A. C. Pigon, Economics of Welfare (First Edition), London: MacMillan, 1920, see especially c. p. 194.

[1b] Frank Knight, "Some Fallacies in the Interpretation of Social Costs," Quarterly Journal of Economics, 1924, p. 582.

[1c] Herbert Mohring and J. Hayden Body, "Analyzing Externalities: Direct Interaction vs. Asset Utilization Frameworks," (forthcoming).

[1d] This tax like any charge for a public good should be levied in such a manner that it does not affect behavior. An exception arises when water quality is supplied contingent on location downstream by quality demanding activities. In this case, a franchise tax contingent on this location would induce quality enjoying firms to take into account the increased costs to upstream firms due to the quality demanders' location decisions.

In addition, economic optimization may also require investment in dams weirs, aeration devices and the like to augment the river's waste assimilative capacity.[2] These activities impose costs on the river authority. Efficiency will, in general require the simultaneous optimization of both pollution charges and the level and composition of this investment. But, the larger the investment in the river's waste assimilative capacity, the less "scarce" are its services and the lower are the efficient prices attached to them. Without further evidence and analysis, there is no way to predict the net financial outcome for the river authority.

That financial balance and Pareto - efficiency are both likely to be important for public policy can be demonstrated by two familiar examples. The conflict between efficiency and financial balance in the case of public utilities is well known. Efficiency requires a price equal to marginal cost, yet, because of scale economics, this price will not generate revenues sufficient to cover total costs. Another example constantly reappears in the public finance literature: the efficient marginal tax rate on incomes or goods is zero, but such taxes fail to transfer resources away from economic agents to the Government. If it should be that a positive surplus is likely to accrue to the river authority (even without an equity-inspired tax on water quality enjoyers), then problems of subsidy from general tax revenues evaporate--indeed, a positive surplus would be an ideal

[2] Indeed, water quality management plans often assume away the possibility of waste discharge reductions, focusing exclusively on capacity augmentation. See the Davis study (cited below) of the Potomac estuary for a case in point.

non-distorting source of government revenue. This paper presents some evidence that, indeed, a surplus is likely to accrue to such an optimizing river authority.

Section II presents a formal model of a riverbasin designed to illuminate the problem of choice among levels of pollution and downstream water quality Marginal welfare maximization conditions derived from this model form one of the links in the financial balance analysis. Sections IV and V present simple but realistic models designed to derive theorems concerning probable surpluses accruing to water quality management. In brief, I conclude that low-flow augmentation, coupled with an optimal level of pollution charges, will probably generate substantial surpluses. Artificial aeration will probably break even or generate a small deficit. The methods used to derive these conclusions are similar to those used to explain to a class in elementary economics why public utility pricing at marginal cost will lead to a deficit. Knowledge of the shape of relevant cost or product curves is often sufficient to derive quite powerful and important results.

II. Economic Maximization Model for Water Quality and Pollution

The core of the pollution/quality allocation problem can be captured in a formal economic model. All models, from the formal to the ad hoc sort used in everyday thought, choose to isolate certain important factors, ignoring all others. Some of the left out factors will be discussed at the end of this section, hopefully substituting a net gain in expositional clarity for a relatively small loss in rigor. Consider a hypothetical economy with a river, along which are located m firms. Each of these firms produces a common output Y, using as inputs a purchased good X, plus the inherant advantages S of its site. X stands for all mobile resources for the period relevant to the analysis, and S for the immobile resources. For example, if the analysis were short-run, plant and equipment would be a part of S, but for a long run analysis, a part of X. In addition, the firm may benefit from clean water, the level of water quality being denoted by Q, and from discharges of polluting materials from its plant. Z is the amount of pollution removal services consumed by the firm from the river, measured by the quantity of pollutant discharged.

Each firm's production function is thus assumed to have the form:

(1) $Y^j = Y^j (X^j, Z^j, Q^j, S^j)$.[3]

For convenience, the firms are numbered from upstream to downstream locations. The model makes most economic sense when the first derivatives with respect to X, Z, and Q[4] are positive and the cross derivatives with respect to X on the one hand and Z or Q on the other hand are negative. For long run adjustments on the non-fixed portion of S, the same comments as for X apply. X, Z and Q are "goods", i. e., they each have positive marginal products, since discharging less pollutants or putting up with poorer quality water requires increasing the amount of the conventional input, say for increased intake or waste water treatment, if output is to remain constant.

[3]The following notation conventions are used:
Right superscript, for an index denoting a particular firm or location;
Left superscript, index denoting a particular individual;
Subscript, partial differentiation with respect to indicated variable;
Parentheses enclose function arguments, and
Brackets are used for multiplication.

[4]But see below. In some contexts generally accepted quality criteria, defined algebraically so that increases in Q benefit most firms, may represent marginal "bads" to some other firms.

Water quality at each location is assumed to be determined by the amount of pollutants discharged upstream and on the investments made to augment the natural waste assimilative capacity of the river. In matrix notation:

$$Q - \bar{Q} = A(X^i) \cdot Z, \tag{2}$$

where Q and $\bar{Q}$ are respectively $(1 \times (m-1))$ vectors of actual quality and quality in the absence of any pollution, leaving out Q^1, which is not affected by investments or pollution. Z $(1 \times (m-1))$ represents pollutant removal services consumption at the first $(m-1)$ locations, with Z^m a free good since there is no one downstream to be affected by it.

The A matrix establishes the relationship between the upstream pollutant discharges and downstream quality, and its elements are assumed to be non-positive and independent of Z. Investment of resources X^I in waste assimilative capacity augmentation is assumed to reduce in absolute value at least some of the negative elements of A, and the second derivatives with respect to X^I all assumed to be positive. Capacity augmentation activities reduce the harmful effects, at the margin, of pollutant discharges on downstream firms, but there are diminishing returns to this activity. These assumptions as to the properties of equations (1) and (2) are sufficient to insure that the first order welfare extrema conditions derived below for river oriented production of Y are indeed maxima. Empirical evidence supporting them is given in Sections III, IV.

Finally, there are <u>n</u> individuals possessing convex ordinal utility functions of the form.

(3) $\qquad {}^iU = {}^iU\,({}^iX,\ {}^iY),$

in which both X and Y are "goods" entering with positive marginal utilities. Social Welfare, the assumed maximand of the model, is assumed to depend on the utilities of each individual consumer:

(4) $\qquad W = W({}^1U,\ {}^2U,\ {}^3U,\ .\ .\ .\ .\ .\ .\ .\ ,{}^nU)$

The usual assumption is made that each individual counts; that is, that $W_i > 0$ for all i. W, like the ${}^iU$'s is an ordering relation with arbitrary units and origin. Its important substantive assumption is that, if the consumption of each individual is known in two alternative states B_1 and B_2, then it is possible to state whether B_1 is "better" , "worse" or "the same" as B_2.

The goal of the model is the maximization of W, as constrained by the endowment of X, the sets of production relations (1) and (2), and identities insuring that the amounts of X and Y allocated to all economic units match the available supply:

(5)
$$\text{Max}\, W^* = W({}^1U\,({}^1X,\ {}^1Y),\ {}^2U\,({}^2Y,\ {}^2X),\ \ldots,\ {}^nU({}^nX,\ {}^nY))$$

$$-\sum_{j=1}^{m} \alpha^j\ [\,Y^j - Y^j\,(X^j,\ Z^j,\ Q^j,\ S^j)\,]$$

$$+\sum_{j=2}^{m} d^j\ [\,(Q^j - \sum_{k=1}^{j-1} a^{jk}\,(X^I)\cdot Z^k)\,]$$

$$+L\ [\,(\bar{X} - X^I - \sum_{i=1}^{n} {}^iX - \sum_{j=1}^{n} X^j)\,]$$

$$+G\ [\,(\sum_{j=1}^{m} Y^j - \sum_{i=1}^{n} {}^iY)\,]$$

Equation (5) contains 4 m + n - 1 variables[5] and 2 m + 1 constraints[6], leaving 2 m + n - 2 degrees of freedom. If the (m-1) Z^i and X^I are specified, then the Q^i are determined. This plus the m X^i suffice to determine the amount of both X and Y available for consumption. If any (n-1) ${}^iX$ or ${}^iY$ are known, the n^{th} amount is determined by the available supply.

Differentiating W* with respect to each of the variables and setting the result equal to zero leads to the following seven sets of 4m + n - 1 equations:

(6) $$W_i \, {}^iU_x = L \qquad (i = 1, \ldots, n)$$

(7) $${}^j \, Y^j_x = L \qquad (j = 1, \ldots, m)$$

(8) $$W_i \, {}^iU_y = G \qquad (i = 1, \ldots, n)$$

(9) $${}^j = G \qquad (j = 1, \ldots, m)$$

(10) $${}^j \, Y^j_z = \sum_{h=j+1}^{m} a^{hj} \qquad (j = 1, \ldots, m-1)$$

(11) $${}^j \, Y^j_Q = - {}^j \qquad (j = 2, \ldots, m)$$

(12) $$- \sum_{j=2}^{m} \alpha^j \sum_{k=1}^{j-1} a_I^{jk} \, Z^k = L$$

[5]The variables are the m Y^i, m X^i, (m-1) Z^i, (m-1) Q^i,

n ${}^jY$, n ${}^jX$, and ${}^iX$

[6]The constraints are the m firm production functions incorporated by the Lagrangian multipliers ^{i}, (m-1) river services relationships incorporated by the ^{i}, and two adding-up constraints incorporated by L and G.

The Lagrange multiplier L has a straightforward economic interpretation, as it is the first derivative of W* with respect to the resource base $\overline{X}$. Since the units and origin of W* are arbitrary, it is convenient to select a scale such that L 1, so that W* can be measured in units of X. This particular scale has all of the ordinal properties of any of the others, but, of course, no cardinality implications. One interpretation of X and W* under this scale which fits the circumstances of the model is to call X "dollars". Under this interpretation, $P_x = L = 1$. The multiplier G is similarly interpreted as the marginal constrained welfare of the produced good Y.

The W_i in equations (6) and (8) are the weights given in the welfare function to increments in individual's utility. A meaningful discussion of real wealth distribution would require a bit more in the way of institutional trapping. However, since:

$$\frac{U_y}{U_x} = \frac{G}{L}$$

is behaviorally consistent with $G = P_y$ and $L = P_x = 1$, G can be interpreted as the market price of Y in terms of numeraire X. "The price of Y" is synonymous with "the marginal social benefits of Y, given the distribution of real wealth (i. e., utility income)." Without loss of generality, G and L may be replaced respectively by P_y and P_x.

From (7) ,

(7)' $$^{j} = P_x / Y_x^j \quad , \qquad (j = 1, \ldots, m).$$

is the marginal cost of Y in welfare units; under the selected scale with $P_x = 1$, it is the marginal cost of Y in terms of the numeraire X. Equation

(7) states that, as one of the necessary conditions for welfare maximization, marginal costs in all firms must be equal, so each ^{j} may be replaced with a common . Equation (9) then takes on the familiar "price equals marginal cost" interpretation.

Equations (10), (11) and (12) state formal conditions for water quality management activities. Even if the other optimizing conditions were not met, they would still represent "second-best" solutions, provided only that the management activities themselves did not contribute to the suboptimization elsewhere.[7] From (10) and (11) substituting P_y for ^{j}, we have:

$$(13) \qquad P_y Y^j_z = P_y \; {}^{m}_{h=j+1} \quad Y^h_Q \quad a^{hj} \qquad\qquad (j = 1, \ldots, m-1)$$

Equation (13) states that, whatever the state of waste assimilative capacity of the river represented by the a^{hj} elements, pollution discharges at any point <u>j</u> ought to be expanded until their marginal returns in production are just offset by marginal downstream sacrifices in production.

Substituting equations (9) and (11) into (12) and noting that the second summation of (12) is $Q^j \quad X^I$, we have:

$$(14) \qquad P_y \; {}^{m}_{j=2} \quad Y^j_Q \quad Q^j_I = P_x$$

X^I should be increased until the marginal benefits from X due to increased

[7]This is a stringent proviso. For example, river services allocation activities will have implications for at least the distribution of real wealth, in the absence of automatic costless wealth redistribution institutions.

quality, given the vector of pollutant discharges, equals the price of X.[8]

So much for the formal optimization conditions, which will be used later. What about the simplifying assumptions which the model inevitably makes? It can be argued intuitively that many changes which would make the model "more realistic" (and more complex and difficult to interpret) would not be likely to change the meaning of the formal conditions for optimum water quality management.

1. Water quality desired for its own sake, rather than as a factor of production.

   Equations (3) would be re-written as:

   (3^1) $\quad {}^iU = {}^iU\,({}^iX,\ {}^iY,\ Q^1,\ \ldots,\ Q^m)$, $(i = 1, \ldots, n)$, with at least some ${}^iU_{Q^j} > 0$. Equations (13) and (14) would then have additional terms reflecting the value of quality to individuals directly. The pricing and allocation problems common

[8] Alternatively, the river services relationship could have been written in inverse form as:

$$Z = A^{-1}\,[\,Q - \bar{Q}\,]$$

A sufficient condition for A^{-1} to exist is that all diagonal elements of A be non-zero. Without exploring the subject in detail, if a diagonal element $a^{j+1,j}$ is equal to zero, then it's also likely that a^{kj}, $(k > j + 1)$ are also zero, making Z_j a free good. The inverse relationship can be interpreted as follows: Z^1 can be inferred from the change in Q^2 from its natural state. Z^2 can be inferred from the change in Q^3 and Q^2; both Z^1 and Z^2 affect Q^3 but noting the change in Q^2 allows the inference of Z^1 and hence the subtraction of its effect on Q^3. Proceeding by induction, knowledge of Q allows one to infer Z. In this case, the benefits from X^I would appear as the algebraic sum of the marginal net benefits from increased pollution, holding Q constant. Some elements of Z, of course, may be reduced at the margin by an increment in X^I.

to all public goods would be increased in magnitude but not in kind.

2. Two types of river-oriented production.

   More than one type of output is in fact produced using water resource services. Industrial water users or municipal water and sewer systems place a higher value on pollutant removal services and a lower value on water quality than do boaters, fishermen, bathing beaches and the like. Expanding the model to include y different types of production would not change the formal conditions for river services allocation. However, sub-optimum allocation of river services would have implications for the distribution of wealth via the differing equilibrium mix of consumption, given the pattern of resource ownership and head taxes.

3. Several firms discharge pollutants or enjoy quality at the same location.

   From the point of view of the river, aggregate pollution discharge should be considered as one pollution source in the model. In addition, marginal benefits from this service should be equal for all firms, strengthening the information - economy argument in favor of pollution charges instead of direct regulation of discharges. The benefits from water quality to each enjoyer would appear in the summations in (13) and (14).

4. Q harmful to some river users.

Perhaps apocryphal tales are told wherein studies have indicated negative net benefits from quality improvements, as when increased dissolved oxygen causes dock pilings to rot faster or reduced thermal pollution causes navigation lanes to be blocked with ice. Conceptually, any public good can have negative marginal value to some people; certainly this is the case for at least military defense policy. If the algebraic sum of weighted downstream Y^j_Q is negative, then of course public policy requires expansion of upstream Z's beyond the free good level of comsumption. None of the formal conditions change.

5. Multiple pollutants or quality parameters.

If there is no interaction among pollutants while in the river, then the conditions governing each separately would be identical to those involving Z. Interaction would require a more complex river services function than (2), but would otherwise introduce no new principles. For multiple Q parameters, (13) and (14), would involve summation both over locations and over types of pollutant.

6. A and Z not independent.

On a formal level, the coefficients a^{jk} in the optimization equations would be replaced with the more general $\partial Q^j / \partial Z^k$

In fact, conservative pollutants, DO/BOD, and perhaps thermal pollution as well satisfy tolerably the independence assumption. To the extent that the independence assumption is true the practical difficulties in calculating optimal pollution discharges and water quality are reduced. See also the discussion in Section VI of the DO/BOD relationship.

7. Multiple-purpose riverbasin investments.

Many waste assimilative capacity augmentation activities have other outputs as well. A dam may provide storage for low - flow augmentation, navigation, and/or flood control. These outputs may be either substitutes or complements at the margin. In terms of the formal model, the increment in X^I can be treated as the marginal expenditure on capacity augmentation, minus the algebraic sum of other marginal benefits from this activity. The likely effects of multiple use on the meaning of the empirical evidence on the shape of relevant marginal cost and product curves is ignored in this paper.

III. Conservative Pollutants and Low-Flow Augmentation

If the river's services arc allocated *via* the price mechanism, an income is generated equal to the sum of the pollutant charges collection at each site.[9] Clearly, if no expenditures are made to augment the river's waste assimilative capacity, a substantial income would accrue to the water resource management authority. If such resources are invested, then the river's capacity is enlarged, and the optimum vector of prices would be reduced. Revenues will increase only if demand is relatively elastic, but not as rapidly as the quantity of services sold. If demand is relatively inelastic, revenues from pollutant charges will actually decrease. Could it be that the income from an optimal level of pollution charges would fall short of paying the costs of an optimal level of capacity augmentation? It can be shown that for a class of simple but realistic examples that this unfortunate financial result is not very likely.

Consider first the simple case of a conservative pollutant discharged by Firm 1, which adversely affects downstream Firm 2. The concentration of this substance at the downstream location (assuming no other sources of this pollutant) is given by:

$$z^2 = Z^1/F, \tag{15}$$

where F is the river's flow at the downstream location. The greater the concentration z^2, the poorer the quality of water there. It is customary in economics

[9]Since a site tax to reflect benefits from water quality should, if properly specified, have no allocational efficiency implications, it will be ignored in this section.

to define goods and services so that more of the commodity is better than less. One convenient scale defines Q^2 by:

(16) $$Q^2 \quad z^* - z.$$

This scale of measure is negative for values of z above the standard, positive for cleaner water, and reaches its maximum of value of z* when the substance is completely absent. The change in Q^2, using this notation, is:

(17) $$\begin{aligned} Q^2 - \bar{Q}^2 &= z^* - z^2 - z^* \\ &= - z^2 \\ &= - Z^1/F \end{aligned}$$

In this simple case, the optimizing equations[10] reduce to:

(18) $$P^1_Z = P^2_Q \frac{\partial Q^2}{\partial Z^1}$$

(19) $$P_F = P^2_Q \cdot \partial Q^2 / \partial F$$

where $P^1_z = P_Y \; \partial Y^1 / \partial Z^1$ and $P^2_Q = P_Y \; \partial Y^2 / \; Q^2$ and P_F = marginal cost of additional flow. The demand for pollutant removal services is given by:

(20) $$Z^1 = d^1(P^1_Z)$$

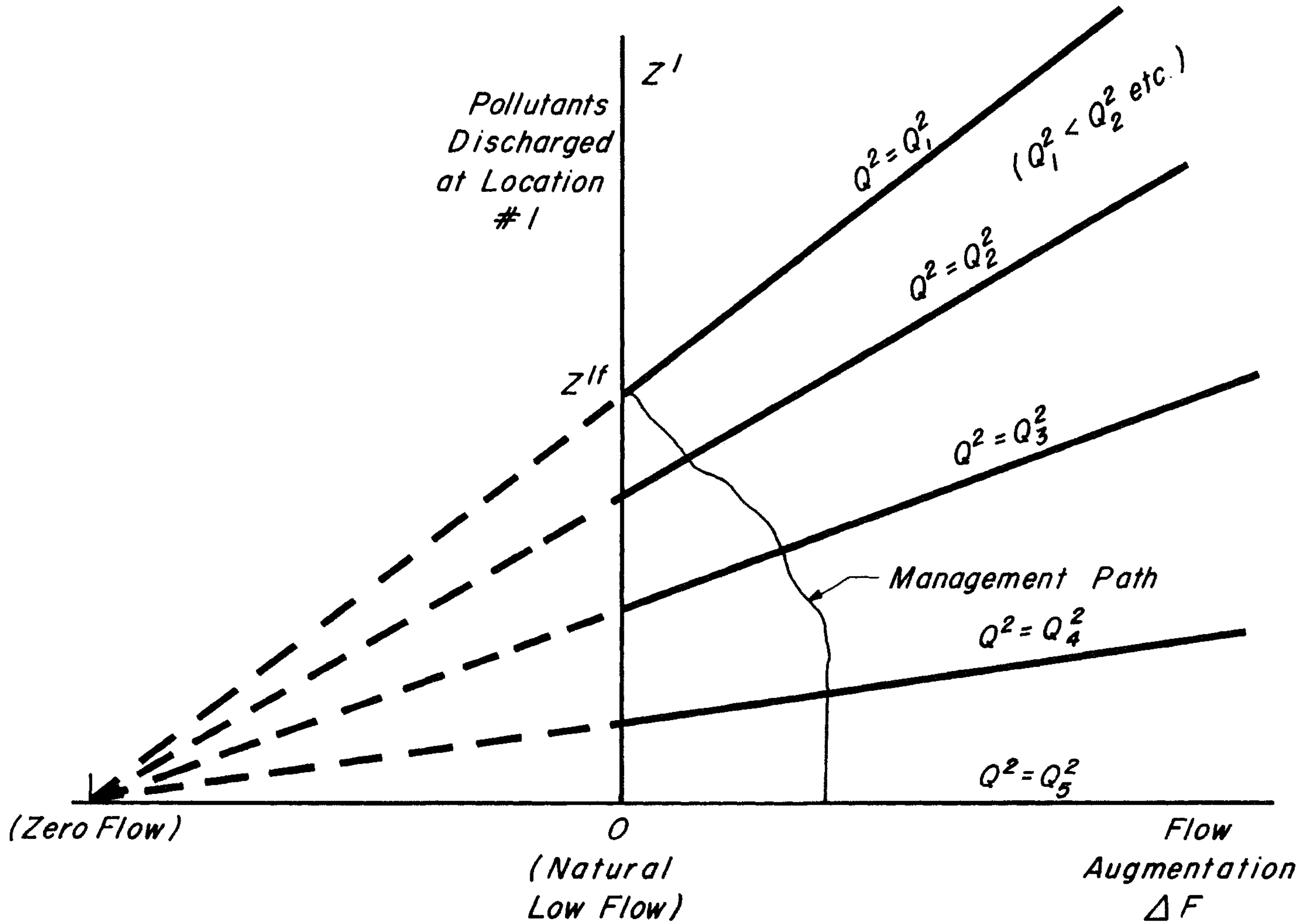
At $P^1_Z = 0, \; Z^1 = Z^f > 0;$ at $Z^1 = 0_1 \; P^1_Z = P_2^{max} > 0$

and d^1 is assumed to have the usual negative slope.

Figure 1 graphs this simple system. Combinations of flow augmentation and pollution discharges lead to levels of quality as shown by the isoquality lines.

10 See equations (13) and (14), Section II.

Figure 1. Isoquality Map, Conservative Pollutant, Low Flow Augmentation



From equation (17) , these isoquality contours, projected to the left, converge to the horizontal axis at a level of ΔF equal to the negative of the natural flow.

Equations (18) and (19) together define the least-cost condition for achieving any level of quality:

(21) $$\frac{P_F}{P^1_Z} = - \frac{\partial Q^2/\partial F}{\partial Q^2/\partial Z^1} \equiv - dZ/dF$$

For $P_Q = 0$, the appropriate $P_Z = \Delta F = 0$, and $Z^1 = Z^f$. For larger values of P_Q, P_Z increases, Z decreases, and ΔF increases when the higher Q is achieved via a least - cost path. This locus of least-cost combinations of Z and ΔF to achieve given levels of Q is analagous to the conventional firm's expansion path and it can be called the "water resource management path." Since Z^1 is likely to go to zero before P^1_Z goes to infinity, the management path will probably have a vertical segment intersecting the horizontal axis.

The relationship between pollution charge revenues and flow augmentation costs is easily derived. Let $\bar{F}$ = natural flow and ΔF = level of flow augmentation. then:

(22) $$Q^2 - \bar{Q}^2 = -Z^1 / (\bar{F} + \Delta F)$$

For any level of $Q^2 - \bar{Q}^2$ (i. e., any value of P_Q) such that pollution reduction is called for, equation (21) reduces to:

(23) $$P_F/P_Z{}^1 = Z^1/(\bar{F} + \Delta F)$$
$$\Delta F = P^1_Z \; Z^1/P_F - \bar{F}$$

If the marginal cost of low flow augmentation, P_F, is a constant then the surplus is calculated by:

$$(24) \qquad R = P_Z^1 Z^1 - P_F \Delta F = P_F \bar{F}$$

In words, the natural flow will earn a per-unit rent just equal to its marginal product. If P_F is an increasing function of F, then the surplus will be larger than $P_F \bar{F}$, smaller if P_F is declining. The combination of a fairly small natural flow combined with relatively large economies of scale in flow augmentation could produce a deficit. An argument is presented below suggesting that P_F is more likely to be rising than falling, strengthening the conclusion that a surplus is likely from optimal water quality management policy.

The extension of the analysis to more than one upstream polluter is straightforward. Let firms 1 through (m-1) have given demands for the conservative pollutant removal service, and let firm <u>m</u> be the only one benefiting from reductions in z concentration. Then, it can be shown that:

$$(25) \qquad Q^m - \bar{Q}^m = - \sum_{j=1}^{m-1} Z^j / (\bar{F} + \Delta F),$$

and the optimal flow augmentation is:

$$(26) \qquad \Delta F = \sum_{j=1}^{m-1} Z^j P_Z / P_F - \bar{F}.$$

The marginal effect of the pollutant is independent of where it is discharged, so that $P_Z^j = P_Z$ for all <u>j</u>. The surplus accruing to the water resource authority is given by:

$$(27) \qquad R = \sum_{j=1}^{m-1} Z^j P_Z - P_F \Delta F = P_F \bar{F},$$

for the case in which P_F is constant.

Now, let us consider the general case of m firms, each one of which may have demands for both Z and Q. The optimization relations, from (13) and

(14) of Section II, can be written:

(28) $$P_Z^j = -\sum_{h=j+1}^{m} P_Q^h \frac{\partial Q^h}{\partial Z^j} \qquad (j = 1, \ldots, m-1)$$

(29) $$P_F = \sum_{j=2}^{m} P_Q^j \, \partial Q^j / \partial F$$

As before,

(30) $$Q^j - \overline{Q}^j = -\sum_{k=1}^{j-1} Z^k/(\overline{F}^j + \Delta F), \qquad (j = 2, \ldots, m) \; ,$$

(31) $$\partial Q^h / \partial Z^j = -1/(\overline{F}^j + \Delta F), \qquad (h = 2, \ldots, m)$$

(32) $$\partial Q^j / \partial F = \sum_{k=1}^{j-1} Z^k / (\overline{F}^j + \Delta F)^2$$

where $\overline{F}^j$ is the natural flow at location j and ΔF is the flow augmentation, assumed equal for all locations.

Substituting equations (31) into (28) and solving for the P_Q^j, we have:

(33) $$P_Q^m = P_Z^{m-1} / (\overline{F}^m + \Delta F),$$

(34) $$P_Q^j = (P_Z^{j-1} - P_Z^j) / (\overline{F}^j + \Delta F), \qquad (j = 2, \ldots, m-1).$$

Substituting these values for P_Q^j in (29) yields:

(35) $$P_F = \sum_{j=1}^{m-1} \left[\left(\sum_{k=1}^{j} Z^k\right) \cdot \left(P_Z^j - P_Z^{j+1}\right) / \left(\overline{F}^{j+1} + \Delta F\right) \right]$$

Let $\overline{F}$ min = min $(\overline{F}^j)$. Then,

$$P_F \leq \sum_{j=1}^{m-1} \left[\left(\sum_{j=1}^{j} Z^k\right) \left(P_Z^j - P_Z^{j+1}\right) / \left(\overline{F}^{min} + \Delta F\right) \right]$$

(36) $$P_F \leq [1/(\overline{F}^{min} + \Delta F)] \sum_{j=1}^{m-1} P_Z^j Z^j$$

If all $\overline{F}^j$ are equal, then the equality holds.

Equation (36) implies that the surplus from management activities will be at least as great as the equivalent expressions derived earlier:

(37) $$R = \sum_{j=1}^{m-1} P_Z^j Z^j - P_F \Delta F > P_F \overline{F}^{min}$$

Time constraints did not permit a detailed investigation of the cost of flow augmentation. Elements which create a presumption that the marginal cost function is increasing can be briefly sketched, however. Simulations using synthetic hydrology indicate that expected yield per unit of storage capacity declines with capacity, ceteris paribus. [11]The unit cost of storage, it may be argued, exhibits the conventional, textbook "U" shape. Particularly for narrow, "V" shaped valleys, increasing dam height increases reservoir capacity more than dam costs. After some point, the valley may flare out at the top, requiring major increments in dam costs per increment in storage capacity. Depending on the contour of the valley, the amount of innundated land per unit of capacity may also be expected to increase after some point.

This line of reasoning is consistent with a finite number of dams on a river system as a minimum - cost system of flow augmentation. Increasing degrees of flow augmentation should imply increases in the optimal number of dams, if

[11] Myron B. Fiering, "The Nature of the Storage-Yield Relationship," Symposium on Stream-Flow Regulation, Robert A. Taft Sanitary Engineering Center, Cincinnati, Ohio, June, 1965, p. 243. See especially Table 1, p. 249.

unit cost curves are indeed "U" shaped. It is worth noting that the Corps of Engineers plans for low-flow augmentation for quality control on the Potomac Estuary do involve multiple dams, with more dams indicated for more low-flow augmentation.[12]

12 Robert K. Davis, The Range of Choice in Water Quality Management: A Study of Dissolved Oxygen in the Potomac Estuary (Johns Hopkins Press for RFF; forthcoming).

IV The BOD-DO Relationship, Low-flow Augmentation, and Artificial Aeration

The importance of dissolved oxygen content (DO) as a water quality criterion, and its dependence on upstream discharges of oxygen demanding materials,(biochemical oxygen demand or BOD) is well known in the engineering literature. Streeter and Phelps[13] are generally credited with the discovery of the basic principles underlying this relationship, an exposition of which can be found in many sanitary engineering textbooks[14]. The basic assumptions underlying the Streeter-Phelps model are that (a) the rate at which dissolved oxygen is utilized is directly proportional to the amount of BOD remaining at any time and (b) the rate of atmospheric reaeration is proportional at any time to the difference between saturation and actual concentration of dissolved oxygen at that time. Thus, both the BOD and DO present at any given time (at any given point in space, if flow rates are known) may be expressed as solutions to the following differential equations:

$$\frac{dL(t)}{dt} = K_1 L(t) \quad (38)$$

$$\frac{dD(t)}{dt} = K_1 L(t) - K_2 D(t), \quad (39)$$

where:

[13] Streeter, H. W., and E. B. Phelps. A Study of the Pollution and Natural Purification of the Ohio River. III. Factors Concerned in the Phenomena of Oxidation and Reaeration P. H. Bull. No. 146, Feb., 1925.

[14] An excellent exposition is given in Louis Klein, Aspects of River Pollution, New York: Academic Press, Inc., pp 147-153

$L(t)$ = BOD remaining at time t (ppm).

K_1 = deoxygenation constant (= .23/day at 20^{o} C),

$D(t)$ = DO deficit at time t

= [saturation value of DO (ppm)] - [actual value of DO (ppm)]

K_2 = reaeration constant ($\cong$ .12 to 1.2).

The model in Section II suggests a division of the river into (m + 1) segments bounded by the m sites. Let L^i and D^i denote respectively the BOD and DO concentration at the i th site immediately upstream from its wastewater outfall. Then the BOD concentration just downstream from site i is given by $L^i + Z^i/F^i$, where Z^i is the physical quantity of wastes discharged and F^i the river's flow at i.

For the simplest case of a single waste outfall at location i, equations (38) and (39) integrate to:

(40) $$L = L_a e^{-K_1 t}$$

(41) $$D = D_a e^{-K_2 t} + L_a \frac{K_1}{K_2 K_1} [e^{-K_1 t} - e^{-K_2 t}]$$

where $L_a = Z_i/F_i$, the BOD concentration immediately downstream of site i, and D_a = the beginning DO deficit. Equation 41 is often called the "oxygen sag" equation, for the obvious reason that D reaches a maximum for a certain downstream t, thereafter declining asymptotically to zero.

The Streeter-Phelps relation has the property that any D^i is a linear combination of upstream Z^i. Let $L^1 = D^1 = O$ and define:

(42) $$\alpha^i = e^{-K_1^i t^i}$$

$$(43) \qquad \gamma^i = e^{-K_2^i t^i}$$

$$(44) \qquad \beta^i = K_1^i/(K_2^i - K_1^i) \quad [e^{K_1^i t^i} - e^{K_2^i t^i}]$$

$$= K_1^i/(K_2^i - K_1^i) \quad [\alpha^i - \gamma^i]$$

where t^i, K_1^i, and K_2^i are the transit time and deoxygenation and reaeration constants appropriate to the river segment between site $\underline{i - 1}$ and site $\underline{i}$. Then,

$$(45) \qquad L^i = \sum_{j=1}^{i-1} Z^j \; [\prod_{k=j+1}^{i} \alpha^k/F]$$

which is a linear combination of upstream BOD discharges, describes the BOD concentration just upstream of the $\underline{i}$ th waste outfall.

The corresponding expression for D^i is somewhat more complex, although it is simple to prove that D^i has the form:

$$(46) \qquad D^i = \sum_{j=1}^{i-1} C^j Z^j$$

where the C^j are constants. Recognizing that the oxygen deficit just downstream from site (i-1) at the beginning of the $\underline{i}$ th river segment is the same as D^{i-1}, equation (41) can be written:

$$(47) \qquad D^i = D^{i-1} \gamma^i + L_a^{i-1} \beta^i$$

$$= D^{i-1} \gamma^i + [L^{i-1} \beta^i + (Z^{i-1}/F) \beta^i]$$

Clearly the last two terms in (47) are a linear combination of the Z^j ($j < i$). If, in addition, D^{i-1} is a linear combination of the upstream Z^i, then, by the associative law of addition, D^i must also be such a linear combination. $D^1 = 0$ by hypothesis, so that D^2 has this property, therefore, by mathematical induction, all D^i are linear combinations of upstream pollutant discharges. The C^j in equation (46) are of the form E^j/F, where the E^j are combinations of the α, β, γ constants:

$$D^i = \sum_{j=1}^{i-1} (E^j/F)\, Z^j \tag{48}$$

That downstream DO is a linear function of upstream pollutant discharges is convenient, since it reduces information requirements for river services optimization. It is interesting to note that more complex models of rivers or even of estuaries which incorporate oxygen sinks or sources, eddy currents, tidal flows and other physical phenomena, still preserve this important additivity property.[15]

Low-flow augmentation will, of course, increase F in equation (48). If that were the only effect, the analysis could proceed along the lines of Section III's argument, and exactly the same conclusion would emerge: Given constant P_F, an optimal level of ΔF and an optimal vector of P_Z, then the authority would reap a surplus equal to the marginal-product rent on the natural capacity of the river.

[15] Richard Frankel, Economic Evaluation of Water Quality: An Engineering - Economic Model for Water Quality Management. SERC Report No. 65-3, University of California, Berkeley; Robert V. Thomann, "Mathematical Model for Dissolved Oxygen", Journal of the Sanitary Engineering Division, ASCE, Vol. 89, No. SA5 (Oct. 1963). Leo J. Hetling, "Water Quality Model of the Estuary", Appendix A in Robert K. Davis, op. cit.

Aside from increases in dilution, low-flow augmentation may be expected to (1) increase the speed of transport, reducing t^i for each segment, (2) change the reaeration coefficient K_2; and (3) change the initial conditions, namely D^1 and (through effects on changes in temperature) K_1 and K_2. The partial effect of (1) will be to shift the low point of the oxygen-sag curve downstream. K_2 increases with speed of flow and decreases with the ratio of surface to volume, so that the net effect of flow augmentation on this parameter is indeterminant.[16]

Water discharged from reservoirs will be less than fully saturated with DO. In general, water drawn from the surface will be of better quality than that from near the bottom of the dam.[17] Water releases may be artificially areated, by a variety of means, each involving some cost.[18]

The effect of low-flow augmentation on downstream water quality is uncertain, in the absence of further information specific to the river in question. A crude weighting of the factors enumerated above creates the presumption that low-flow augmentation is likely to decrease D, given Z.

[16] See O'Conner, D. J. and W. E. Dobbins, "The Mechanism of Reaeration in Polluted Streams", J. of the Sanitary Engineering Division, ASCE, 82: (SA6): Paper 1115, 1956; and, Churchill, M. A. et al. "The Prediction of Stream Reaeration Rates". op. cit. (SA4), 1962, pp. 1-46.

[17] R. A. Vanderhoof, "Changes in Waste Assimilation Capacity Resulting from Streamflow Regulation", pp 139-140. Symposium on Stream-Flow Regulation for Quality Control, Robert A. Taft Sanitary Engineering Center, Cincinnati, Ohio, June, 1965.

[18] See, for example, Averill J. Wiley et al.., "Commercial Scale Stream Reaeration", J. of Water Pollution Control Federation, April, 1962, p. 401, for an analysis of introducing oxygen into hydroelectric generating turbines.

There is evidence that the state of engineering knowledge is somewhat deficient with respect to the prediction of the effects of flow augmentation. An extensive survey of the literature conducted by the Public Health Service found that "very little has been written on the influence of impoundment releases on downstream water quality, and those articles that have been prepared are concerned with the degradation of downstream water quality because of releases of poor quality water from impoundments ... No discussions were found concerning the beneficial effect that might accrue from the discharge of good quality water."[19] Nevertheless, the Corps of Engineers has prepared multi-million dollar plans involving the construction of dams and reservoirs whose main function is the improvement of water quality, principally DO levels downstream from sewage outfalls. One presumes that the Corps is satisfied that it is able to successfully predict the total effect of low-flow augmentation on downstream DO levels.

We turn next to some scraps of evidence as to the shape of the marginal-product-of-flow function. Hetling[20] has presented results of simulation model calculations for the Potomac estuary, excerpted in Table 1. The flows reported are those required to maintain a given minimum level of DO throughout the estuary, as a function of temperature and organic loading from municipal sewage

[19] James M. Symons et al., Influence of Impoundments on Water Quality. A Review of Literature and Statement of Research Needs U. S. Dept. of HEW, PHS, Division of Water Supply and Pollution Control, October, 1964, pp 49-50.

[20] Hetling, Op. cit, Table 1.

at Washington, D. C. The isoquality map for a temperature of 30° C is depicted in Figure 2.

The condition for achieving a given water quality at minimum cost is, from equation (13) of Section II,

$$(49) \qquad \left. \frac{\partial UOD}{\partial F} \right|_{\Delta DO = O} = P_F / P_Z$$

Table 1. Flow Requirements For Given Oxygen Target, Waste Load, and Temperature

DO = 4 ppm

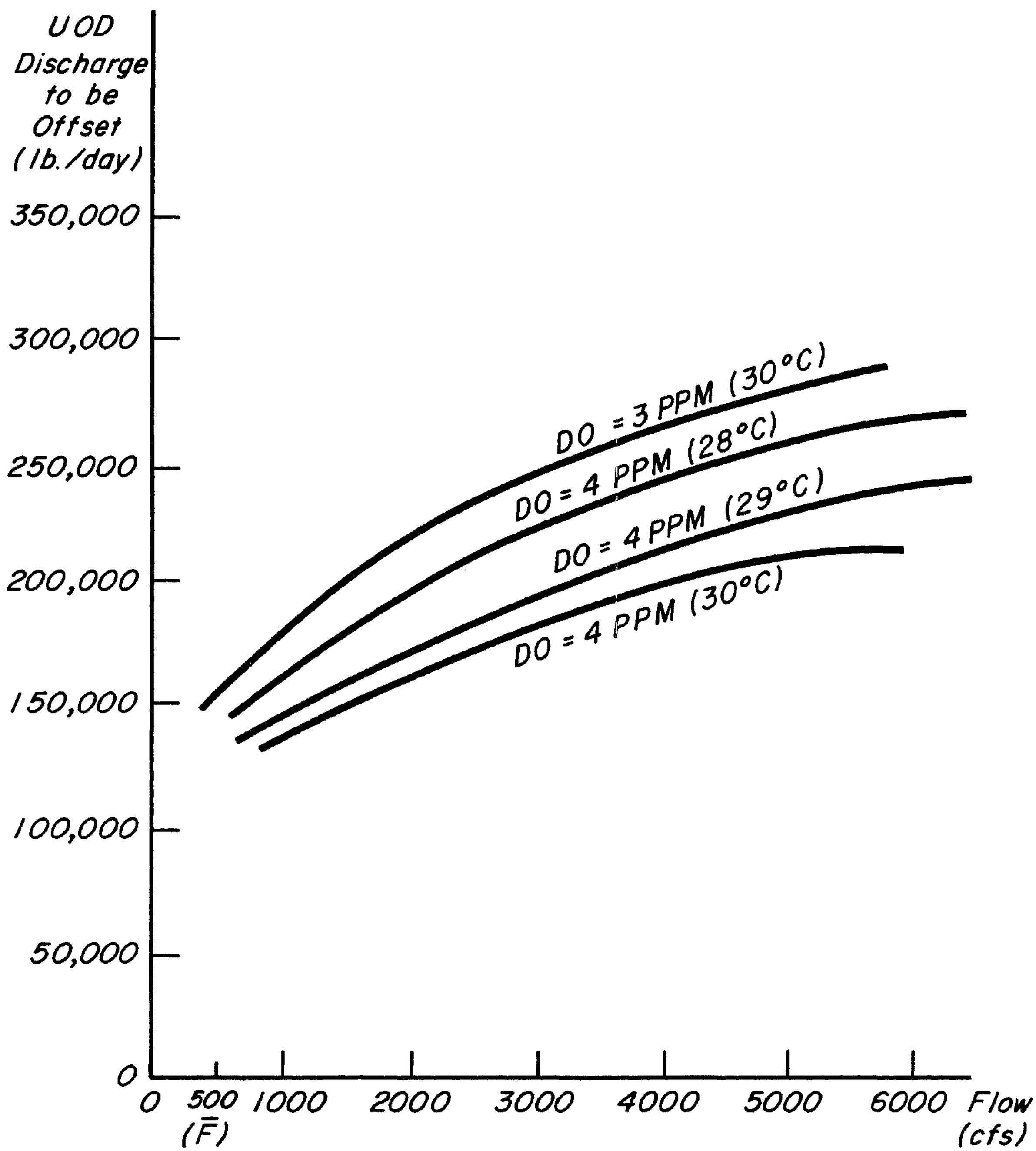
Temp. =	28°C			29°C			30°C		
UOD (lb.)	Flow (cfs)	ΔUOD / F	UOD / F	Flow (cfs)	ΔUOD / F	UOD / F	Flow (cfs)	ΔUOD / F	UOD / F
140,000	*500		280	800		175	1100		127
		35.3			22.2			16.2	
200,000	2200		91	3500		57	4800		42
		15.8			13.3			18.8	
260,000	6000		43	8000		33	8000		33

DO = 3 ppm

Temp. =	30°C		
UOD (lb.)	Flow (cfs)	ΔUOD / F	UOD / F
140,000	*500		280
		60.0	
200,000	1500		133
		24.0	
260,000	4000		65

*Required flow less than or equal to 500 cfs.

Figure 2. Flow Requirements, Potomac Estuary



From (49) we see that the surplus accruing to the river authority is (assuming constant P_F) :

$$(50) \qquad R = ZP_Z - \Delta F\, P_F = P_F\ (Z\ \frac{UOD}{F} - \Delta F),$$

where ΔF is the level of flow augmentation. R is positive if and only if:

$$(51) \qquad Z/\Delta F > \partial UOD/\partial F.$$

Inspection of Table 1. indicates that equation (51) is strongly satisfied. Even under the extreme assumption that $\Delta F = F$, $Z/\Delta F$ is declining, with the marginal product of flow $\partial UOD/\partial F$ far below the average product.[21] The surplus is greater, therefore, than $(\bar{F} + \Delta F)\ P_F$ rather than equal to this magnitude as for the conservative pollutant considered above. The presence of a surplus may also be inferred from Figure 1., since a tangent to the isoquality curve intersects the X axis far to the left of the origin, even farther to the left of any positive natural low flow.

We turn next to artificial aeration as a means of quality improvement. Susag _et al._[22] present experimental data on mechanical aeration of the Mississippi River downstream from St. Paul, Minnesota. Table 2 summarizes data from their Table II. The authors proffer no causal relationship between oxygen

[21] In his calculations, Hetling assumed a base (i.e., naturally occurring minimum) flow of 500 cfs.

[22] Russell H. Susag, Robert C. Polta and George J. Schroepfer, "Mechanical Aeration of Receiving Waters," _Journal of the Water Pollution Control Federation_, January, 1966, p. 53.

transfer and prevailing conditions, although their equations and text hint strongly that turbine horsepower per unit of flow is a relevant parameter.

It happens that the observed data can be cast reasonably well into an exponential model resembling the solution to the Streeter-Phelps relationship. Specifically, I hypothesize that:

(52) $$D_1 = D_o e^{-kA}$$

where D_1 = DO deficit downstream from the turbine, D_o = DO deficit upstream from the turbine, and A = turbine power per unit of flow. Solving for K,

(53) $$k = \frac{\ln D_o - \ln D_1}{A}$$

Table 3 indicates that k exhibits a reasonable stability across observations.

The placement of the aerators on the oxygen sag curve should depend on the demand for water quality at different downstream locations. Obviously, the oxygen transferred per horsepower will be greatest if the device is located at the bottom of the sag, but this would be indicated only if oxygen content were most valuable, at the margin, at the downstream of this point. The determination of optimization rules for turbine placement should be reasonably straightforward, but beyond the scope of this paper.

Assume that there is one turbine downstream from a BOD outfall and one quality-demanding location _i_ immediately downstream from the turbine. From equation (46) above:

$$D^i = C^j Z^j \qquad (j < i),$$

where C^j is a constant, given the conditions of flow, temperature and the like.

TABLE 2: Effect of Mechanical Aeration on Dissolved Oxygen Levels

Observation	KW	Flow	DO Upstream mg/1.	DO Downstream mg/1.	DO Added
1	.625	1.14	4.35	6.30	1.95
2		.81	2.30	6.30	4.00
3		1.14	.05	5.25	4.20
4		.54	1.20	7.10	5.90
5		.93	4.55	7.35	2.80
6		1.95	7.00	7.65	.65
7		.88	.20	5.30	5.10
8		1.95	1.75	4.35	2.60
9		1.98	2.25	4.55	2.30
10		3.70	6.00	6.50	.50
11		3.60	2.05	3.45	1.40
12	.630	1.11	2.15	5.70	3.55
13		2.10	1.90	3.90	2.00
14	.620	3.67	.40	1.85	1.45
15	.625	3.55	1.35	2.60	1.25
16	.615	2.16	1.30	3.35	2.05
17		.96	.70	4.75	4.05

Source: Susag, et al., Table II

TABLE 3: Calculated k Parameter Under Exponent

Observation	A KW/cfs	D_o (ppm)	Log D_o	D_1 (ppm)	Log D_1	Log D	Hypothesis k	1/k
1	.548	4.82	.683	2.87	.458	.115	.411	2.433
2	.771	6.87	.837	2.87	.458	.379	.492	2.032
3	.548	8.12	.910	3.92	.593	.317	.578	1.730
4	1.157	7.97	.901	2.07	.316	.585	.506	1.976
5	.672	4.62	.665	1.82	.260	.405	.603	1.658
6	.321	2.17	.336	1.52	.182	.154	.480	2.083
7	.710	8.97	.953	3.87	.588	.365	.514	1.945
8	.320	7.42	.870	4.82	.683	.187	.584	1.712
9	.316	6.92	.840	4.62	.665	.175	.554	1.805
10	.169	3.17	.501	2.67	.427	.074	.438	2.283
11	.174	7.12	.852	5.72	.757	.095	.546	1.831
12	.567	7.02	.847	3.47	.540	.307	.541	1.848
13	.300	7.27	.862	5.27	.722	.140	.467	2.141
14	.169	8.77	.943	7.32	.865	.078	.462	2.164
15	.176	7.82	.893	6.57	.818	.075	.426	2.347
16	.285	7.87	.896	5.82	.765	.131	.460	2.173
17	.641	8.47	.928	4.42	.645	.283	.441	2.267

Source: Table 2.

Let D^{i*} be the deficit downstream from the turbine at location $\underline{i}$, so that, from (12):

$$D^{i*} = D^{i} e^{-kA} = C^{j} Z^{j} e^{-kA}$$

Solving for Z^j:

(54) $$Z^{j} = \frac{D^{i*} e^{kA}}{C^{j}}$$

Holding D^{i*} (and, hence, Q^{i}) constant, this is the equation for isoquality contours.

As before, the quality - supply - function condition implies that:

(55) $$\frac{P_A}{P_Z} = \frac{\partial Z^{j}}{\partial A} = k \quad Z^{j}$$

The surplus accruing to the river authority is, on the assumption of constant P_A;

(56) $$R = ZP_Z - AP_A = P_A \ (1/k - A).$$

Inspection of Table 3. indicates that 1/k is around four times A for the experimental turbine used by Susag, et al. Assuming constant P_A and no other form of capacity augmentation, if cost minimization called for installation of a turbine meeting the specified experimental conditions, then the authority would achieve a surplus.

The authors present cost data suggesting that fixed charges are on the order of two-thirds of total costs, operating and maintenance making up the balance. Electricity is usually available at a lower unit cost in large quantities, and there may be scale economies in maintenance also. No parametric cost data for the turbines themselves were presented. However, the extended-aeration, activated sludge waste treatment technology involves an aerating turbine in a tank, so it is possible that stream reaeration turbines involve the same kind of scale economies. Published data indicate that the capital costs of an extended aeration plant may be estimated by a relationship of the form:

(57) $$C = kS^{\alpha},$$

where k and α are parameters and S is system size.[23] Values for α range from .5 to .65, indicating substantial scale economies. However, a large portion of these scale economies may stem from the tank itself. For example, if the tank is a cube of side L, the costs are proportional to the surface area or L^2 and system capacity to volume L^3, then $C = kS^{.67}$.

To the extent scale economies do exist, P_A in equation (55) will be a declining function of A, so that

(57) $$R < ZP_Z - AP_A = P_A\ (1/k - A).$$

Evidence from the Potomac River study suggests that R is likely to be, on balance, slightly negative. Further evidence on the shape of the reoxygenation cost curve is provided by Davis.[24] Table 4 summarizes for, two types of reoxygenation devices, costs which would be incurred to offset four levels of

[23]Herbert Mohring and J. Hayden Boyd, Economics of Water Use in Petroleum Refining, Chapter V (Forthcoming); see also Appendix below.

[24]Davis, *op. cit,* Chapter V.

pollutant discharge at Washington, D. C. In each case, the quality criterion was 4mg/1 minimum monthly mean DO. The range of costs represents uncertainty about the performance and costs of given equipment; however, the cost ranking of these and other alternative systems was not very sensitive to the width of the cost range. Note an apparent anomaly in the minimum estimates for mechanical reoxygenation, in that costs listed for 30,000 and 60,000 are greater than for 90,000 and 120,000 lbs. UOD offset.

Costs per pound of UOD decline with increasing UOD, and marginal costs are below average costs, but the difference is not great. The earlier conclusion of at least mild scale economies thus receives additional confirmation.

Finally, Table 5 reports estimates of least-cost combinations of techniques for achieving alternative levels of DO, assuming 120,000 lb/day UOD discharge. That a minimum level of low flow augmentation is optimal for each quality goal and further increases in quality are achieved by increasing reoxygenation is consistent with the earlier evidence of sharply increasing costs for the former and mildly decreasing costs for the latter technology.

The implications of the foregoing for the probable relationship between pollution charges income and the costs of augmenting the river's waste assimilative may be quickly summarized. The maintained hypothesis, as before, is that the river authority selects a vector of P_Z and a level of capacity augmentation which minimizes the social costs of attaining a given level of water quality. The presence of increasing costs for flow augmentation, coupled with a "free" natural flow, indicate that a substantial surplus would be generated for this activity. Conversely, pollution charges just high enough to cover the costs of

TABLE 4: Estimated Costs for Two Kinds of Aeration Devices, Potomac Estuary[1]

UOD Offset (lb/day)	Cost ($$10^6$ Present Value) Low	High	Cost/UOD[2] ($/(lb/day))	Δ Cost/Δ UOD[2]
Diffused Reoxygenation				
30,000	2.2	12.0	400	
				200
60,000	3.3	18.0	300	
				300
90,000	4.6	27.0	300	
				267
120,000	5.9	35.0	292	
Mechanical Reoxygenation				
30,000	4.4	9.2	307	
				260
60,000	7.7	17.0	283	
				267
90,000	3.9	25.0	278	
				233
120,000	4.7	32.0	267	

1. Source: Davis, op. cit., Table 9.

2. Average and marginal costs based on "High" cost estimates.

TABLE 5: Costs of Achieving Alternative Dissolved Oxygen Objectives in Potomac Estuary ($$10^6$ present value)

DO mg/1	Low Flow System	Least Cost Alternative[1]
2	8	15
3	27	18
4	115	22
5	∞	27

Source: Davis, *op. cit.*, Table 17.

1. Includes $6 million for minimal low-flow augmentation

flow augmentation would be too low. On the other hand, reaeration costs are fairly close to constant, so that financial balance is roughly consistent with optimization. A combination of these techniques, assuming that the cost of the combination is lower than either alone, would lead to a surplus representing rents accruing to flow, with rents accruing to reaeration being essentially zero.

Appendix: Parametric Cost Functions for Oil Separators, Activated Sludge and Trickling Filters.

I Technology and Costs of Waste Treatment

For obvious reasons, the cost of waste treatment is a fundamental determinant of the demand for pollution removal services. There is wide agreement that any degree of pollutant reduction of any waste flow is possible using conventional equipment designs, although the associated marginal cost is an increasing function of the amount of pollutants removed. The rational firm, when faced with a vector of prices for the various pollution removal services, will seek to bring the corresponding vector of marginal costs of waste treatment into equality with them. Capital and operating costs of waste treatment facilities will be examined, in order to develop the relevant long-run[1] marginal cost functions for waste treatment.

Forbes and Witte[2] present graphs relating capital costs of waste treatment facilities to volume of wastes to be treated. Table 1 translates these graphs into algebraic form. In each case, an influent BOD of 200 ppm is assumed for these costs. The authors maintain that costs are proportional to influent BOD concentration, implying capital cost functions of the form:

[1] That is, incremental costs, with respect to important parameters, of plants designed for alternative performance levels.

[2] M. C. Forbes and P. A. Witte, Philosophy, Methods and Costs of Refinery Waste Disposal, National Petroleum Refiners Association, Tech. 65-19, June, 1965.

TABLE 1: Costs of Waste Treatment Facilities

Capital Costs

$C = aF^{\alpha}$, where C = installed cost in $\$10^3$, F = flow in 10^6 gal/day.

Facility	a	α
Pretreatment	\$ 46.3	.90
Packaged Flotation Units	28.6	.47
Packaged Coagulation Plants	.435	.90
Imhoff Type Plants	365.	.83
Primary Treatment-Separate Sludge Digestion Plants	274.	.55
Stabilization Ponds	80.5	.58
Activated Sludge Plants	380.	.77
Activated Sludge-Prater and Antonacci	127.	.47
Activated Sludge-USPHS	379.	.61
Trickling Filter-Separate Sludge Digestion Plants	341.	.56
Trickling Filter-Imhoff Type Plant	312.	.68
Trickling Filter-J. B. Dannenbaum	192.	.70
Trickling Filter-USPHS	328.	.70

Operating Costs

$C = a\ F^{\alpha}$, where C = operating costs in $\$10^3$/yr.

Maintenance Costs	198.	.99
Total Operating Cost, Including Chlorination	16.6	.88
Operating Labor Costs	10.0	.86

Source: Forbes and Witt.

$$C = \frac{B}{200} \cdot a F^{\alpha}$$

where C is capital cost in $\$10^3$, B is the strength of the raw wastes in ppm, F is the rate of waste water flow in 10^6 gallons per day, and a and α are parameters from Table 1. The authors caution that, "These figures will allow the determination of the order of magnitude only, . . . (although) the data have been cross checked between sources and related to actual installations of our own experience where possible."[3] Another source states that this kind of data is "probably no more than 60% accurate."[4]

Crude or not, this data suffers from the additional defect that no information is given regarding the associated removal efficiencies. Prater and Antonacci[5] do state, however, that their data refer to plants designed to achieve 85-95% BOD reduction. It should also be noted that assuming costs proportional to BOD concentration, given declining unit costs with respect to volume of flow, implies that capital costs could be reduced indefinitely by diluting the raw wastes with fresh water! For these reasons, it was decided to examine the basic physical and biochemical mechanisms of several waste treatment techniques, in order to estimate how costs might vary as the degree of waste reduction is varied.

[3] _Op. cit._, p. 13.

[4] Milton R. Beychok, _Aqueous Wastes from Petroleum and Petrochemical Plants_, New York; John Wiley and Sons, 1967, p. 276.

[5] _Op. cit._, p. 150.

Detail findings for oil separators, activated sludge plants, and trickling filters will be presented in turn. In addition, other treatment technologies will be described briefly.

II The Economics of Oil-Water Separation

The separation of oil from waste water makes use of the fact that oil and water have different specific gravities. Oily water standing in a container, if the oil is not emulsified, will separate into three layers, most of the oil rising to the top, and a small part settling to the bottom as sludge. Modern oil separators are essentially devices which accomplish this separation on a continuous flow basis.

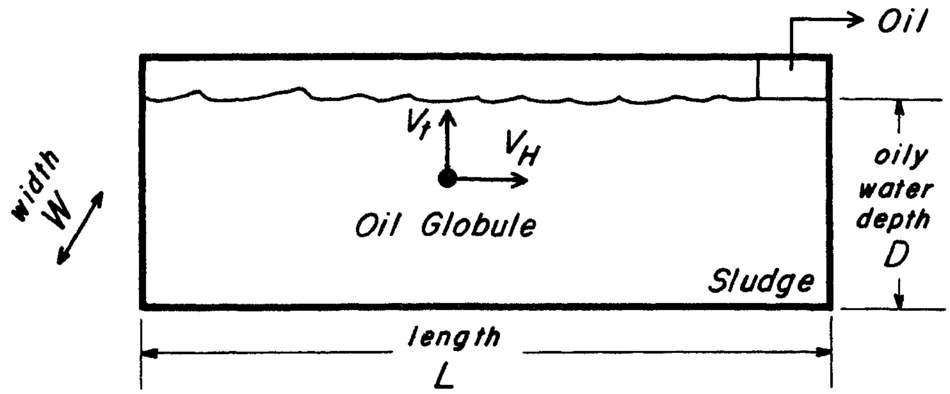
The first API Manual on the Disposal of Refinery Wastes, concerned with the design and construction of oil separators, was published in 1934. This manual is currently in its seventh edition.[6] The API separator remains the standard for the petroleum refining industry, although other technologies, described below, have been devised in the last few years.

An API separator, depicted schematically in Figure 1, is essentially a long concrete box. Oily water enters from the left, and flows slowly to the outlet end with horizontal velocity V_H. Oil globules, being lighter than the water, rise to the surface and are collected by a skim pipe and withdrawn. In addition, some heavier-than-water fractions settle to the bottom of the separator, necessitating periodic cleaning.

The rate, v_t, at which an oil globule rises toward the surface depends on the specific gravities of the oil and the water and on the globule's diameter.

[6] API, Manual on the Disposal of Refinery Wastes, V. 1, Waste Water Containing Oil, 1963.

Figure 1. API Oil Separator



The upward force acting upon a globule is proportional to the difference in specific gravities of oil and water, times the volume of the globule. The frictional force resisting this upward force is roughly proportional to the cross section area of the globule. Hence, smaller globules rise more slowly than larger, and the separator effluent will contain a preponderance of smaller globules. The separator design criterion can be expressed in terms of the smallest size globule to be totally removed.

A typical separator design criterion calls for removing all globules 0.015 cm in diameter or larger[7] of an oil of .995 specific gravity. Associated with this kind of design criterion is a design upward velocity V_t*; for the particular criterion above V_t* equals 0.278 ft./min. If all globules in this size class are to rise to the surface during the time the oily water spends in the separator, the

7 Op. cit., p. 22.

ratio of separator depth to length must be no greater than the ratio of V_t^* to V_h:

(1) $$\frac{D}{L} \leq \frac{V_t^*}{V_h}$$

The horizontal velocity, in turn, depends on the rate of flow of oily water, Q, and on the cross sectional area of the separator:

(2) $$V_h = Q/W\,D$$

Substituting for V_L in equation 1 and rearranging,

(3) $$V_t^* \geq \frac{Q}{WL}$$

The term on the right of equation (3) is known as the "overflow rate." This relationship states that the overflow rate in an ideal separator can be no larger than the upward velocity of the globules the separator is designed to remove. If the separator exactly meets design criteria, then the equality will hold. Stated differently, an "ideal" API separator will remove all globules whose upward velocity is greater than the overflow rate.

If it were literally true that depth had no influence on separator performance, then the minimum cost separator would consist of a square slab with infinitesimally high sides. In fact, the API design criteria indicate certain minimum and maximum values which are recommended for separator depth and width and the ratio of depth to width, as well as a maximum recommended horizontal velocity. Associated with these limits are correction factors for turbulence and short circuiting. The design tolerances and the associated correction factors were apparently derived on the basis of a tradeoff between the losses in oil removal performance due to further departures from ideal separator performance and those due to reductions in overflow rate as separator dimensions are altered.[8]

The specific recommended limits are:

$D = 3 - 8$ ft.
$W = 6 - 20$ ft.
$D/W = 0.3 - 0.5$
$V_h \leq 15\ V_t$
$V_h \leq 3.0$ ft./min.[9]

The associated correction factor, K for turbulence and short circuiting is 1.55. Applying this correction factor to equation 1,

(4) $$\frac{D}{L} = \frac{V_t^*}{V_h} \cdot \frac{1}{K} = \frac{V_t}{V_h} \cdot \frac{1}{1.55}$$

Equations (2) and (4) allow the dimensions of the separator to be expressed in terms of Q, V_t^*, V_h and the ratio D/W. Let D/W = X. Then, equation (2) can be re-written:

(5) $$W^2 = \frac{Q}{XV_h}.$$

Furthermore,

(6) $$D = WX = \sqrt{\frac{QX}{V_h}}.$$

[8] Op. cit., p. 20.

[9] Op. cit., p. 21.

From equation (4) ,

$$(7) \qquad L = K \frac{V_h}{V_t^*} D = K \frac{V_h}{V_t^*} \frac{QX}{V_h} .$$

Assume that costs are proportional to the surface area of the sides, ends and bottom of the separator, and write

$C = c(2DL + 2DW + LW)$.

Substituting and simplifying, this expression becomes

$$C = c \left[\frac{2Q}{V_h} + (2X + 1) \frac{K Q}{V_t^*} \right].$$

Costs are clearly a decreasing function of V_h and an increasing function of X. Economical design requires therefore that the maximum permissible horizontal velocity be used, and that the ratio of depth to width be set equal to .3, provided that this X does not lead to the violation of the restrictions on D and W.

For X = .3 and D = 3, W must equal 10, and the rate of flow, 90 ft./min. or 673.2 gal/min. For smaller flows, keeping D at its minimum permissible value of 3 ft., the ratio of depth to width must be increased. The smallest flow consistent with suggested design limits is 54 ft.^{3}/min. or 403.92 gal./min. For larger flows than 673 gal./min., costs per gallon of oily water flow should be constant until the maximum recommended W of 20 ft. is reached at 360 ft.^{3}/ min. or 2693 gal./min. In fact, the height of the separator wall above the level of liquid remains constant for all flows, and the depth of the separator and hence its cost ought to increase somewhat less than proportionally with flow over the range 673 - 2693 gal./min.

If the separator is built with two or more bays, it will be possible periodically to divert the oily water flow away from each bay in order to clean it. Since very little cost penalty is associated with this benefit, one would expect multiple-bay separators to be the rule. This does indeed seem to be the case.[10]

The above cost behavior predictions are in accord with the fragmentary cost data available in the literature. Beychok[11] shows unit costs declining sharply over the range 200-500 gal./min. and much more slowly thereafter. Forbes and Witt[12] indicate that costs of pretreatment may be estimated as

$$C = 46.3\ F^{.90} \tag{8}$$

where C is capital cost in thousands of dollars and F is the rate of flow of oily water, expressed in millions of gallons per day.

Cost estimation equations such as (8) presuppose some given oil removal performance. In order to estimate the changes in costs associated with changes in removal performance, additional information is needed. The theory underlying oil separator design and expressed in equation (4) suggests that decreasing the overflow rate is necessary to achieve greater separation. In other

[10] See e.g., W. L. Pursell and T. W. Ferguson, "New Oil-Reclamation and Disposal Facilities," API Division of Refining, Proceedings, Vol. 34 (III), 1954, p. 190; E. D. Newman, et al., "Waste Disposal at Anacortes," Oil and Gas Journal, May 19, 1958, p. 126.

[11] Beychok, op. cit., p. 277.

[12] M. C. Forbes and P. A. Witt, op. cit. This source is cited by Beychok for separators in the over-500-gal./min. class.

words, the smaller the overflow rate of an oily water with a given size and specific gravity distribution of oil globules, the larger is the proportion of these globules which will rise to the surface in the separator. Inspection of equations (5), (6) and (7) indicates that V_t*, which, by equation (4) is equal to the overflow rate, appears only in the expression for L. Therefore, if it is assumed that the bays of the separator are of such a size that the cost minimizing D/W of .3 is maintained, then variations in the overflow rate can be achieved only by varying the length of the bays. The cost of the separator for a given Q may be expected to be proportional to its length, and hence, to the inverse of the overflow rate.

Brunsman, et al.,[13] present laboratory data relating overflow rate to oil removal from a standardized oily water. The specific relationship found[14] for oily water initially containing 650 ppm oil may be expressed algebraically as

(9) $$O_{PE} = 1550\, V_o^{.76},$$

where O_{PE} is the oil content of the separator effluent in parts per million and V_o is the overflow rate in ft./min.

The typical design conditions mentioned earlier lead to a design upward velocity, V_t*, equal to 0.278 ft./sec., and, correcting for turbulence and short circuiting, an overflow rate, V as follows:

[13] J. J. Brunsman, J. Cornelissen and H. Eilers, "Improved Oil Separation in Gravity Separators," Journal of the Water Pollution Control Federation, January, 1962, p. 44.

[14] Op. cit., Fig. 5, p. 47.

(10) $$V_o = \frac{Vt^*}{K} = \frac{0.278}{1.55} = .179 .$$

Substituting in equation (9), we have

$$O_{PE} = 426 \text{ ppm}$$

for the separator meeting this design standard and handling a standardized oily water influent containing 650 ppm oil. This corresponds to the separation of 34% of the oil in the separator influent.

Since the ultimate aim is a cost function containing as one of its arguments the amount of oil removed from the effluent, several additional assumptions are necessary to tie this information to the cost data mentioned earlier. It will be assumed, first, that the size distribution and specific gravities of oil globules in oily water effluents correspond to that used by the authors in the model separator; and second, that the proportion of oil removed, at a given overflow rate, is independent of the initial concentration.

Revising equation (9) to take account of different oil concentrations, we have

(11) $$O_{PE} = \frac{O_{PI}}{650} 1550 V_o^{.76},$$

$$= 2.38 O_{PI} V_o^{.76},$$

where O_{PI} = oil concentration in separator influent ppm. Multiplying both sides by the rate of flow, we have:

(12) $$O_E = 2.38 O_I V_o^{.76},$$

where O_E and O_I refer respectively to oil in pounds per day in the separator effluent and influent. Solving for V_o,

$$(13) \qquad V_o = .318 \left[\frac{O_E}{O_I} \right]^{1.32}$$

It will be recalled that theory suggest that oil separator costs are proportional to the inverse of the overflow rate and that API design criteria call for a V_o equal approximately .179 ft./min. This suggests a cost function of the form

$$C_S = \frac{.179}{V_o} \quad 46.3 \quad F^{.90}.$$

Substituting (13) for V_o, this becomes

$$(14) \qquad C_S = 26. \left[\frac{O_I}{O_E} \right]^{1.32} \quad F^{.90},$$

where C_S is capital cost in \$$10^3$, O_I and O_E are respectively pounds of oil per day in separator influent and effluent, and F is oily water flow in 10^6 gal./day. Daily separator costs, CD_S, would be C_S times (.200/365), assuming a capital consumption allowance of 20% per annum.

The optimal degree of oil separation depends on the economic environment in which the firm finds itself, as well as the technological possibilities open to it. If the firm does not use a secondary (i.e., biological) treatment plant, then the separator effluent is the plant's effluent. Otherwise, it represents the process water inflow to the biotreater. In the former case the cost to the firm of the oil discharged is the value of the discharged oil, plus the pollution removal fee for that oil along with whatever other pollutants might be contained in the oil. In the latter case, the cost of oil in the separator effluent is the value of the oil, plus the loss due to its effect on the removal performance of the biological unit. Balanced against this cost is the cost of the separator. The oily water effluent also has a value to the firm's operation. Reducing its flow would add to other costs.

The foregoing discussion suggests that the net returns from oily water use and discharge can be written:

(15) $$R = \hat{P}F - CD_S(F, O_I, O_E) - P_o O_E,$$

where R = returns, $\hat{P}$ = shadow price or marginal value, in the firm's operation, per unit of flow, P_o = marginal economic loss per unit of oil discharged, and the other variables are defined as above.

If we assume that O_{PI} is a constant, then (15) can be simplified to:

(16) $$R = \hat{P}F - CD_S(O_E, F) - P_o O_E.$$

This expression is maximized when F and O_E are selected so that $\partial R/\partial O_E = \partial R/\partial F = O$, or when:

(17) $$- \partial CD_S/\partial O_e = P_o, \text{ and}$$

(18) $$\partial CD_S/\partial F = \hat{P}$$

These last two equations express the firm's adjustment to the tradeoff, at the margin, among separator size, oil discharged, and water use in its operation.

Differentiating the cost function (14) with respect to O_E;

(19) $$\frac{CD_S}{O_E} = \frac{-1.32\ CD_S}{O_E}.$$

The marginal cost of additional flow, keeping the same proportional removal rate is:

(20) $$\left.\frac{\partial CD_S}{\partial F}\right|_{\Delta(O_I/O_E) = O} = \frac{.90CD_S}{F}$$

On the other hand, if flow is increased without altering (O_I/O_E), then both O_I and O_E will increase. Holding the concentration of influent oil and the amount of oil discharged constant:

(21) $$\left.\frac{\partial\, CD_S}{\partial F}\right|_{\Delta O_E = O} = \frac{.90\, CD_S}{F} + \frac{1.32\, CD_S}{O_I}\,\frac{\partial O_I}{\partial F}.$$

The last term on the right adjusts for the greater removal efficiency needed to keep discharges constant.

Now,

(22) $$O_I \text{ (lb/day)} = F\,(10^6 \text{ gal/day}) \times 8.34 \text{ (lb/gal)} \times O_{PI} \text{ (parts per million)},$$

so that

(23) $$\frac{\partial O_I}{\partial F} = 8.34\, O_{PI}$$

Substituting (18) and (19) in (17) and combining terms yields:

(24) $$\left.\frac{\partial\, CD_S}{\partial F}\right|_{\Delta O_E = O} = \frac{2.22\, CD_S}{F}$$

In the context of economic optimization, equation (24) and not (20) is the relevant derivative. Note that, from equations (17) and (19), expenditures for oil removal services will be a constant fraction of the cost of the optimal separator, since $P_O O_E = 1.32\, CD_S$.

At 30% removal rate and .2 mgd flow, marginal cost of increased flow is less than 6 cents per thousand gallons; at 60%, 12 cents, and at 90%, 78 cents. The marginal flow cost goes up sharply, of course, as very high removal rates are attempted. For many industrial firms, these magnitudes will be dominated by the other costs of using water, and the quantity of water used will be insensitive, over broad ranges, to the degree of oil separation performed.

No data on the operating costs of API separators were found. However, there is reason to believe that the portion of operating costs which vary with removal performance is likely to be insignificant. The standard API design requires no operator attention, and only periodic cleaning to remove accumulated sludge. Power costs should be nil if, as is usually the case, the separator is downgrade from the refinery process units.

Brunsmann, et al.,[15] have reported a modification of the API design that achieves better separation for a given capital cost. They inserted a series of parallel plates in the separator, inclined upward slightly from the horizontal. These plates, spaced one to four inches apart vertically, intercept the rising oil globules, coalescing them into larger, more quickly rising globules. The authors show that the effective surface area of the separator is increased by the sum of the horizontal projections of the parallel plates, proportionally reducing the overflow rate for a given through-put.[16] The chief drawback of this system is the greater cleaning expense, compared to an equivalent API separator, although precise data on this point are lacking.

Two other methods of separating oil from refinery wastes, air flotation

[15] J. J. Brunsman, J. Cornelissen and H. Eilers, "Improved Oil Separation in Gravity Separators," Journal of the Water Pollution Control Federation, V. 34, No. 1, Jan., 1962, p. 44.

[16] Ibid., p. 47.

and chemical coagulation also deserve brief mention.[17] Air flotation units speed up oil-water separation by first dissolving air in the waste in a pressure tank, then releasing the air-water mixture into a flotation basin. The bubbles then formed float the suspended matter to the surface, where it is skimmed off. Flocculating or pH control chemicals, such as alum, are sometimes added to speed the process.[18] Chemical coagulation, as the name implies, adds chemicals to the waste water to form a gelatin-like floc which adheres to the oil and other suspended matter. The floc agglomerates into a settleable sludge, allowed to precipitate and is then removed.

[17] Both of these methods, like the API and parallel plate separators, remove suspended matter (i.e., oil) from water. They are thus incapable of removing dissolved organics, and cannot replace biotreatment units for this purpose. Of course, some phenols, etc., are removed along with the oil, thereby reducing the load of these pollutants in the biotreatment unit, or in the final effluent if no secondary treatment is employed.

[18] R. N. Simonsen, "Oil Removal by Air Flotation at Sohio Refineries," Proceedings, API Division of Refining, Vol. 42 (III) (1962), pp. 405-406.

III Biological Treatment of Wastes: Description of Technologies and Cost Calculations

Three general classes of secondary waste treatment technologies are commonly employed by industrial plants: activated sludge units, trickling filters and oxidation ponds. Each of these will be described below. Detailed, parametric cost functions for the first two are also derived. (The details of this derivation are contained in Part IV of this Appendix.) These empirical functions may be used to estimate marginal waste treatment costs of typical wastes.

The activated sludge system, of which there are several distinct versions, is widely used. Each type of activated sludge system is based on the same type of bacterial reaction. Although the equipment differs, each is essentially a tank in which the waste water to be treated is mixed with a sludge made up of flocculated bacteria. The unstable organics contained in the water are rapidly absorbed by the gel structure of the bacterial mass which makes up the sludge, then are progressively oxidized. Oxygen can be continually supplied by compressed air streams, mechanical agitators or a combination of these methods.

The rate of reduction of the original organics, as for example, phenols, is quite rapid. One hour's retention time in a complete mixing system,[19] for example, will reduce the amount of the original organics by about 93%. However, this oxidation of the organic content of the original waste results in the synthesis of new bacterial protoplasm. This protoplasm, or the <u>active</u> <u>mass</u> of the sludge,

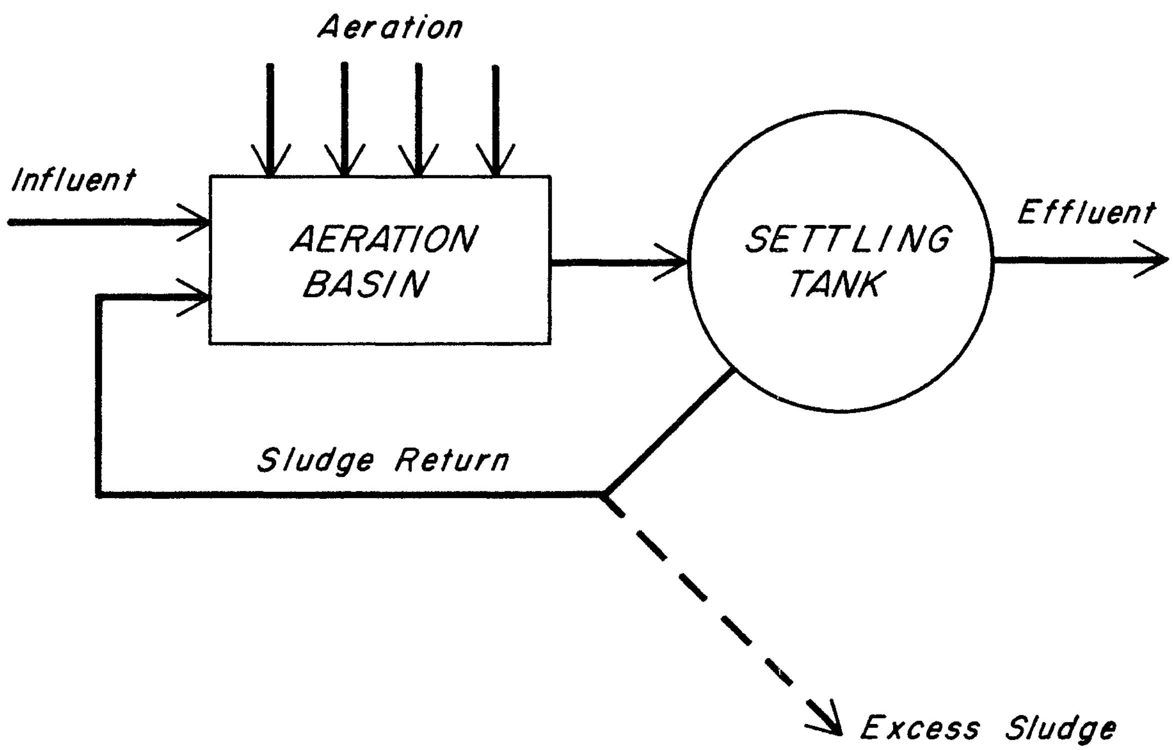
[19] Assuming 200 ppm BOD concentration in the raw waste. See below for the derivation of these removal rates.

is itself an unstable organic which contributes to the BOD of the treatment plant effluent. If the reaction is allowed to continue, endogenous respiration or auto-oxidation will proceed to reduce the protoplasm and its BOD. In effect, a part of the bacteria die and are rather cannibalistically consumed by the remainder. Thus, the same one hour retention time mentioned above will lead to only 55% and 62% BOD reduction respectively for aeration only and aeration with sludge return systems.

The basic activated sludge system is depicted in Figure 2. Raw waste enters the aeration basin where it is mixed with the activated sludge mass and aerated. The mixture then flows from the aeration basin into a settling tank, where the active and inert bacterial mass is allowed to separate for return to the aeration basin. Excess sludge may be either discharged in the effluent or routed to a separate anaerobic sludge digester. Modifications of the basic process include tapered aeration, in which a greater proportion of the aeration is performed at the inlet end of the tank where oxygen demand is the heaviest, and step aeration, in which the influent is introduced at several points along the length of the tanks.

Although versions of the basic system have long been successfully used for the treatment of domestic sewage, their intolerance of shock loads makes them a poor choice for treatment of industrial wastes such as petroleum refinery effluents. When the return sludge and raw wastes are mixed at the inlet end of the aeration basin, the full impact of a shock load is felt by a small portion of the total population of micro-organisms. A heavy shock load of phenols can be bactericidal even to acclimatized bacteria, as can heavy doses of sulfur compounds.

Figure 2. Basic Activated Sludge System



Even when a portion of the active bacteria are not eliminated in this fashion, the microbial population never reaches the relatively constant equilibrium of complete mixing systems.[20] Because of these facts, no attempt was made to develop parametric cost functions for the basic activated sludge system.

As the name implies, complete mixing activated sludge systems differ from conventional systems in that the untreated wastes are practically instantaneously dispersed throughout the aeration tank. The biological population and waste load are thus uniform over the entire tank and it can act as a surge tank to level out variations in the strength of incoming wastes. The theory of complete mixing systems has been developed by McKinney,[21] on whose treatment the following discussion is based.

The simplest complete mixing system is the aeration only system, depicted in Figure 3. The active microbial mass is measured in oxygen-equivalents concentration, or the concentration by weight of the amount of oxygen needed to completely metabolize the protoplasm via endogenous respiration. The total effluent 5-day BOD is equal to 60% of the active mass plus the residual raw organics.[22] Note that the performance of this system, measured in terms of the

[20] Ross E. McKinney, "Mathematics of Complete-Mixing Activated Sludge," Journal of the Sanitary Engineering Division, Proceedings of the American Society of Civil Engineers, May 1962, p. 87.

[21] Op. cit.

[22] All waste load analyses in this study are expressed in terms of 5-day BOD, the oxygen demand for bacterial oxidation of the waste exerted in a period of 5 days at 20° C. The calculations all assume the system to be in steady-state equilibrium.

Figure 3. Aeration-Only, Complete-Mixing System

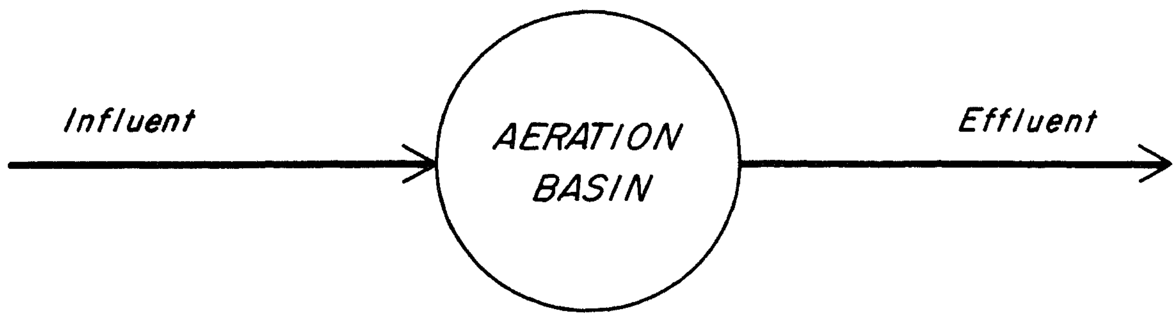


Figure 4. Complete Mixing with Sludge Return

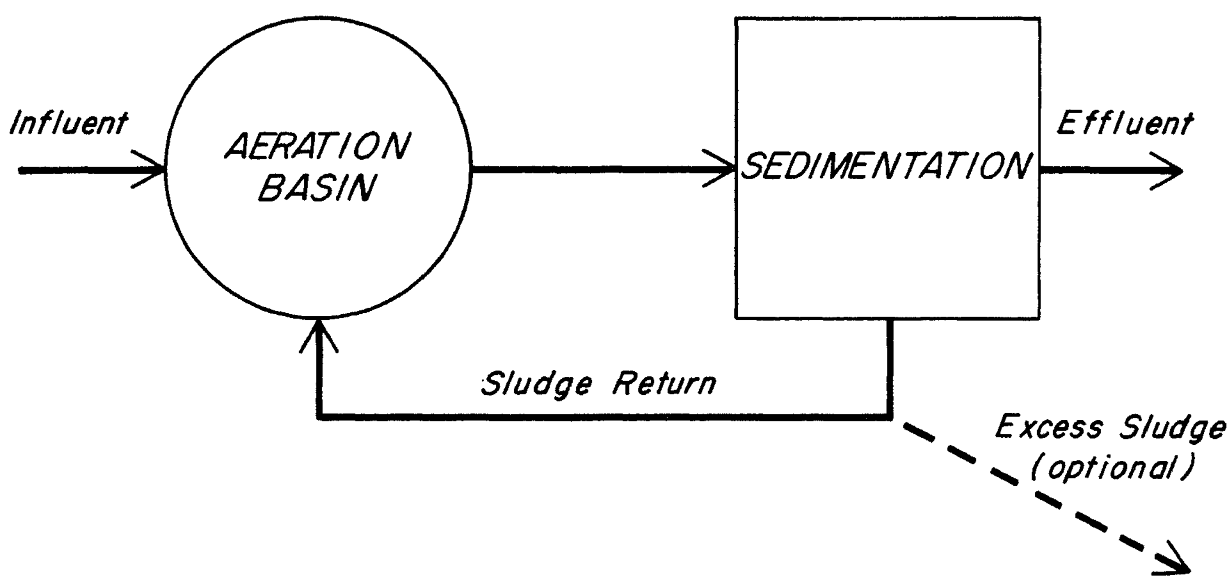
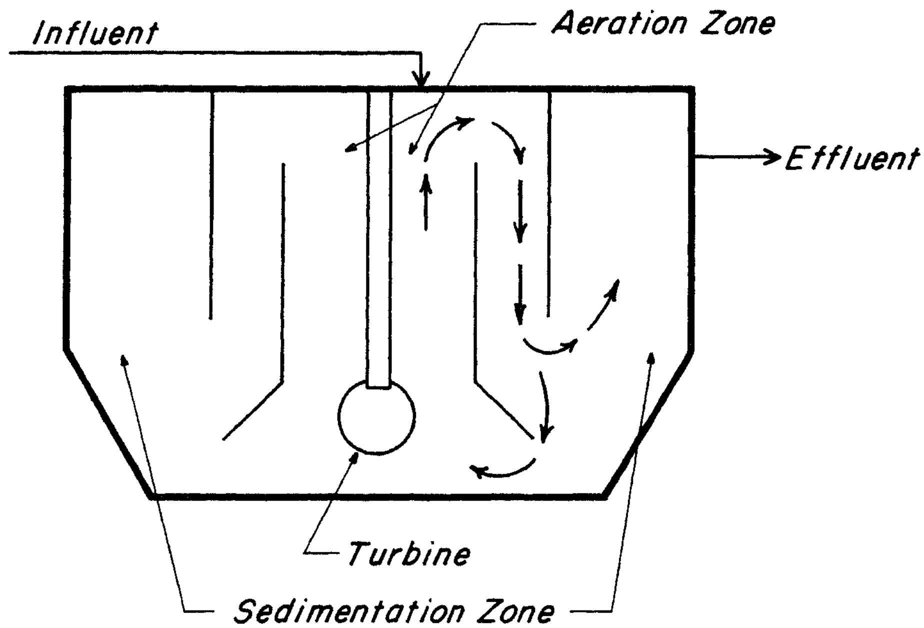


Figure 5. Extended Aeration Version of Complete Mixing, Sludge Return Activated Sludge System.



proportion of either influent BOD or organics removed, is independent of the strength of the raw waste.

Figure 4 is a schematic diagram of a complete-mixing activated sludge system with sludge return. The settling of the effluent allows a higher concentration of active biological mass in the aeration basin, relative to the active mass concentration in the effluent. The higher concentration in the aeration basin allows more rapid oxidation of the raw wastes, and the lower concentration of unoxidized protoplasm in the effluent leads to a nearly proportional decrease in effluent BOD. As with the conventional activated sludge system, excess sludge may either be discharged with the effluent or withdrawn and treated separately in an anaerobic digester.

The extended aeration system depicted in Figure 5 is a version of the sludge return system. It is widely used at pertoleum refineries and in other industrial applications. The excess sludge is discarded in the effluent with this system.

A typical design criteria calls for the recirculation of sufficient excess sludge to permit the total volatile suspended solids, consisting of both active microorganisms and nonliving organics, to build up to 4,000 parts per million.[23] More concentrated wastes are reduced in BOD less rapidly on a percentage basis, although the percent rate of oxidation of the original organics is not affected. The latter rate is also equal to the aeration-only system's performance, and, the rate of BOD reduction can be expected to be higher as long as any sludge at

[23] API, Biological Treatment of Petroleum Refinery Wastes, 1963, p. 52.

all is returned. The less rapid BOD reduction for more concentrated wastes in the sludge-return system reflects the greater amount of excess sludge which must be discharged in order to keep the volatile solids concentration in the aeration basin at the design criterion.

The remaining variable which determines the cost of this treatment system is, of course, the rate of flow of raw wastes. The physical quanitities (by weight) of both influent and effluent pollutants may be calculated by multiplying flow times the respective concentrations, and the size of the aeration tank and other parts of the system, by multiplying flow times retention time.

The equations linking retention time and influent BOD concentration to system performance may be combined with the cost data mentioned above to obtain parametric waste treatment cost function. Prater and Antonnaci's data pertain to costs of a packaged version of the complete mixing, sludge return system discussed above. They state that the design level of performance is for 11 to 14 hours total retention time, yielding 85-95% BOD reduction of wastes initially containing 200 ppm BOD.[24] Using the above relationships, 11 hours retention time leads to 92.6% of BOD reduction, 14 hours to 94.3% reduction, assuming 200 ppm BOD in the raw wastes. For purposes of the following cost calculations, we will assume that the cost data refer to a system designed for a retention time of 12.5 hours. The corresponding BOD and original organics reduction would be 93.6% and 99.5%, respectively, with this influent concentration.

[24] N. H. Prater and D. W. Antonacci, "Find Packaged Sewage Plant Costs", Hydrocarbon Processing and Petroleum Refiner, Feb., 1963, p. 147.

It seems reasonable to postulate that system size, rather than flow *per se*, is the important underlying cost determining parameter. Thus, cost should depend on the product of flow and retention time, rather than on flow. More precisely, the capital cost equations for complete-mixing systems, mentioned above would be transformed into the form:

$$C = a \left[\frac{t}{12.5} F \right]^{\alpha}.$$

This is the basic equation used in the optimization equations reported below.

Operating costs are the sum of power, maintenance and operating labor costs. The U. S. Public Health Service reports power costs for extended aeration plants ranging from \$27 to \$208 per million gallons of actual flow, with an average of \$111.[25] The API estimates power and maintenance costs of about \$284 per million gallons. Labor costs, for operation and analytic control, are estimated to be \$6,000 to \$10,000 per year, independent of flow.[26] It seems reasonable to assume, therefore, that that proportion of power and maintenance costs which varies with retention time is on the order of \$250 per million gallons. Since almost all of the power requirements are for aeration, the variable portion[27] of operating costs will be assumed to be proportional to the rate of oxygen transfer per unit of aeration volume, dO/dt, times the total volume of the system, Ft.

[25] *Op. cit.*, pp. 10-11.

[26] API, *Biological Treatment*..., p. 70.

[27] That is, those costs which are dependent on the retention time and other design variables of the system.

For further information on the procedures used in the derivation of the cost functions, the reader is referred to Part IV.

We turn next to another widely used method of waste treatment, the trickling filter. These devices are simply large tanks, usually cylindrical, filled with stones or a honeycomb of plastic media. The term "filter" is a misnomer, as the BOD reduction is accomplished by bacterial oxidation rather than by filtration. Waste water is distributed over the top of the filter by a rotating boom and allowed to trickle down over the filter media. Bacteria, which adhere to the stones' surfaces or the plastic honeycomb, feed on the organic content of the wastes. Oxygen is provided by simple diffusion as the water is sprayed over the top of the filter.

The absence of separate aeration devices makes the trickling filter less costly to operate than activated sludge units, the API reports power and maintenance costs of only $19 per million gallons. Estimated labor costs are also lower, about $4,000 to $6,000 per year, independent of flow.[28]

Despite the simplicity of this method of biological waste treatment, there is no agreement in the literature as to the parameters which determine trickling filter performance.[29] Galler and Gotaas, using multiple-regression

[28] API, op. cit., p. 70.

[29] A summary of the various theories which have been proferred on this subject is contained in William S. Galler and Harold B. Gotaas, "Analysis of Biological Filter Variables," Journal of the Sanitary Engineering Division, Proceedings of the American Society of Civil Engineers, December 1964, pp. 59-64. See also API, Biological Treatment..., pp. 16-19, 41-46.

techniques to analyze an extensive collection of trickling filter performance data, proffer a relationship of the form:

$$L_e = \frac{K(IL_i + r\,L_e)^{1.19}}{(I+r)^{.78}\,(1+D)^{.67}\,a^{.25}} \tag{29}$$

$$K = \frac{.464\left(\frac{43560}{\pi}\right)^{.13}}{I^{.28}\,T^{.15}}$$

where L_e = effluent BOD (ppm), L_i = influent BOD (ppm), I = raw wastes flow (million gal./day), r = filter effluent recirculated over the filter (million gal./day), D = filter depth (ft.), a = filter radius (ft.), and T = temperature in degrees centigrade.

In order to make the analysis comparable to the design conditions for the activated sludge system, a temperature of 20° C (68° F) was assumed. Then, the effluent BOD concentration, given the characteristics of the waste, can be seen to depend on the filter radius and depth and the amount of recirculation a, D, and r.

In a subsequent article, the same authors present an analysis in which the cost minimizing values of a, D and r are derived for various values of I, L_i and L_e.[30] Since the data on which the regression analysis was based were presumably for filters satisfying certain ad hoc design criteria based on previous research, the authors felt it necessary to establish both upper and lower bounds for the three design variables. (The details of these constraints are reported

[30]Galler and Gotaas, "Optimization Analysis for Biological Filter Design," Journal..., p. 163 ff.

in Part IV, along with the other calculations which were performed.) As a result, the optimal combination of these parameters turned out not to be one which equated the marginal cost of waste reduction with respect to each of them.[31] Rather, it was found that the cost minimizing path, as the degree of removal required was progressively increased, involved increasing filter depth from its minimum to maximum value, then the same for recirculation (which also may force an increase in radius, because the lower bound of a is a function of r), and finally a. If the resulting a is larger than its "maximum value," multiple filters would be required to keep the design parameters within the "acceptable limits."

The present author , using a somewhat higher capital consumption allowance than Galler and Gotaas[32] found that the above path did not minimize treatment costs. The reported calculations are those using an (approximate) equality-of-marginal-cost criterion rather than the path reported by the cited authors.

Even with the revised optimization path, almost all of the resulting designs involved one or two of the three parameters at the a priori upper or

[31] $MC_j = \frac{\partial C}{\partial V_j} / \frac{\partial Z_B}{\partial V_j}$, where MC_j = marginal cost with respect to V_j, V_j = design variable $j(V_1 = a, V_2 = D, V_3 = r)$, C = daily capital and operating costs, and Z_B = pounds of BOD per day in trickling filter effluent.

[32] We used an annual interest-plus-depreciation of 20%, while they used values ranging from 5-15%. Our power cost assumption of 1¢ per KWH was within the range they considered. See op. cit. (1966), p. 169.

lower bound. If equation (25) continues to apply outside these bounds, then lower costs can be achieved by, e.g., increasing r beyond 4 I before a is increased. Unfortunately, further refinement of the economics of trickling filters awaits additional research on the performance of filters whose design parameters lie outside the boundaries assumed by the authors.

The literature seems to contain no information on the fraction of effluent BOD from trickling filters which is in the form of unoxidized original organics, or, conversely, how much is made up of active bacterial protoplasm. This state of affairs reflects, no doubt, the controversy about the quantification of the basic biological mechanisms of the process. According to the API manual, however, at "low" organic loadings, "as the microbial film ages and dies at the stone surfaces, the film drops from the stones and is washed away from the filter. With high organic loadings and high hydraulic loadings the film growth is more rapid; ...the microbial film (washes) from the stone surfaces continually".[33] We may infer, therefore, that at "high" loadings at least part of the effluent BOD is made up of bacterial protoplasm. To the extent that this is the case, the BOD reduction percentages will understate the rate of reduction of phenols and other organics. In the absence of better information, no explicit calculations of phenols removal marginal costs were made.

The third class of biological treatment is the oxidation pond. This method is far more land-extensive than either activated sludge units or trickling filters,

[33] Op. cit., p. 18

and is, therefore, economic only where land is inexpensive. These are shallow ponds, usually arranged in series, and designed for retention times measured in days rather than in hours. Oxygen is supplied mostly by algae which use carbon dioxide and water plus the sun's energy to produce oxygen via photosynthesis. In addition, some oxygen can be provided by diffusion from the atmosphere at the surface of the pond. Because of the extreme variations in land values likely to be found among locations, no attempt was made to develop parametric cost data for this method.

IV Marginal Cost Functions for Biological Waste Treatment

A. Activated Sludge (Complete Mixing Systems):

The basic equations used in these derivations were taken from McKinney.[34] Let L_i = concentration of 5-day BOD in raw wastes in ppm, L^* = the concentration of original organics in the effluent, and t = the retention time in hours. Then,

(26) $$L^* = L_i \,/\, (k_5 t + 1) = L^*(L_i, t),$$

where $k_5 = 15$ /hr. The concentration of active bacterial mass, M_a in ppm oxygen equivalents, is given by:

(27) $$M_a = \frac{k_6 L^*}{1/t + k_7},$$

where $k_6 = 10.42$ /hr. and $k_7 = .006$/hr.

Substituting equation (26) for L^*, (27) becomes:

(28) $$M_a = \frac{k_6 L_i \,/\, (k_5 t + 1)}{(1/t) + k_7} = M_a(L_i, t).$$

The 5-day BOD of the effluent is the sum of the unoxidized remainder of the original organic load and that contributed by the bacterial mass:

(29) $$L_e = L^* + k_{10} M_a = L_e(L_i, t),$$

where L_e = 5-day BOD of effluent, $k_{10} = .6$.

The rate of oxygen demand is given by:

(30) $$dO/dt = k_9 L^* + k_2 M_a = O(L_i, t).$$

[34] Op. cit.

The quality of the effluent can thus be expressed in terms of the raw waste load and retention time. Note that both concentrations L_e and L^* are proportional to L_i.

The complete mixing, sludge return system displays the same rate of L^* reduction as the aeration-only system; see equation (26). Let x be the proportion of M_a which is discharged with the effluent ($O < x < 1$). The active mass can then be expressed as:

$$(31) \qquad M_a = \frac{k_6 L^*}{(x/t) + k_7}$$

The total concentration of suspended solids in the aeration tank, M, is given by:

$$(32) \qquad M = M_a \left(1 + \frac{k_8 t}{x}\right) + M_m + M_i$$

where M_m = metabolized volatile suspended solids (ppm), M_i = ion concentration of inert solids, and $k_8 = .0015/\text{hr}$. Following the API manual,[35] we assume that $M_m = M_i = O$ and that x is chosen to allow M to build up to 4,000 ppm. Thus, equation (29) becomes:

$$(33) \qquad 4,000 = M_a \left(1 + \frac{k_8 t}{x}\right).$$

Equations (31) and (33) suffice to determine unique values of x and M_a in terms of retention time and raw waste concentration. Solving first for x by substituting equation (31) into (33), we have (for the positive root):

[35] API, Biological Treatment of Petroleum Refinery Wastes, p. 52.

(34) $$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = x(L_i, t)$$

where $a = 4,000/t$

$$b = 4,000\ k_7 - k_6 L^* = 4,000 k_7 - k_6 L_i/(k_5 t + 1)$$

$$c = k_6 k_8 L^* t = k_6 k_8 L_i t \ / \ (k_5 t + 1) \ ,$$

M_a may then be calculated by inserting this value of x into equation (31), yielding a function of L_i and t.

As before, the effluent BOD is calculated from the unoxidized organics and that portion of the active mass, xM_a, discharged in the effluent:

(35) $$L_e = L^* + k_{10} x M_a = L_e\ (L_i, t) .$$

The parameters L_i, L_e and L^* are all concentrations, expressed in units of parts per million. Thus, the performance of different size systems depends on retention time, but not on size per se. Let F be the rate of flow of the waste water, in millions of gallons per day. Then 8.34 x F million pounds per day of water will be handled. For each 10^6 pounds/day of water, L_i, L^* and L_e pounds per day of influent BOD, original organics discharge and BOD discharge will be handled. Write, therefore,

(36) $$W_B = L_i\ 8.34\ F$$

$$Z_B = L_e\ 8.34\ F,$$

where W_B and Z_B are respectively BOD loads generated and discharged by the refinery, in pounds per day.

Typically, only a part of L_i is phenols or other original organics. For example, if phenols and other organics are oxidized at the same rate and phenols make up 15% of W_B, then:

(37) $$Z_p = L^* \, 8.34 \; F \; .15,$$

where Z_p is phenols discharged in pounds per day.

The basic design parameter for the activated sludge system is its retention time, t. The economic optimization calculations contain terms involving the rate of change of pollution removal services with respect to this parameter. Differentiating equation (26) we have:

(38) $$\frac{\partial L^*}{\partial t} \quad \frac{-L_i \, k_5}{(\,k_5 t + 1)^2}$$

The equation for active mass is more complex. Rewriting equation (31) symbolically yields:

(39) $$M_a = \overline{M}\,(L^*(t),\; t,\; x)$$

where $x = g(t, L^*)$,

and differentiation yields

(40) $$\frac{\partial M_a}{\partial t} = \frac{\partial \overline{M}}{\partial L^*} \; \frac{\partial L^*}{\partial t} + \frac{\partial \overline{M}}{\partial t} + \frac{\partial \overline{M}}{\partial x}\left[\frac{\partial g}{\partial t} + \frac{\partial g}{\partial L^*} \; \frac{\partial L^*}{\partial t}\right]$$

Taking each term in turn,

(40a) $$\frac{\partial \overline{M}}{\partial L^*} \; \frac{\partial L^*}{\partial t} = \frac{-\,k_6}{(x/_t) + k_7} \quad \frac{L_i \, k_5}{(k_5 t + 1)^2}$$

(40b) $$\frac{\partial \overline{M}}{\partial t} = \frac{k_6 \, L^* \, x}{(x + k_7 t)^2}$$

(40c) $$\frac{\partial \overline{M}}{\partial x} = \frac{-k_6 \, L^*}{t(\,(x/t) + k_7)^2}$$

Write, from equation (34),

$$x = U^{-1} V$$

where $U = 4000k_7 - k_6 L^*$

and $V = k_6 k_8 L^* t - \frac{4000}{t} x^2$.

Differentiating implicitly, we have:

$$\frac{\partial x}{\partial t} = \frac{V k_6}{U^2} \frac{\partial L^*}{\partial t} + (k_6 k_8 (t \frac{\partial L^*}{\partial t} + L^*)$$

$$- \frac{8000x}{t} \frac{\partial x}{\partial t} + 4000\, x^2 / t^2) / U$$

(40d)
$$\frac{\partial x}{\partial t} = \frac{\frac{V k_6}{U^2} \frac{\partial L^*}{\partial t} + \frac{k_6 k_8 (t \frac{\partial L^*}{\partial t} + L^*) + 4000 \frac{x^2}{t^2}}{U}}{(1 + \frac{8000x}{tU})}$$

The rate of change of effluent BOD is, from equation (35):

(41)
$$\frac{\partial L_e}{\partial t} = \frac{\partial L^*}{\partial t} + k_{10} \left[x \frac{\partial M_a}{\partial t} + M_a \frac{\partial x}{\partial t} \right] .$$

As before, these derivatives may be converted from concentration units into physical units. For BOD,

(42)
$$Z_{Bt} = 8.34 \; F \, \partial L_e / \partial t$$

and, for phenols

(43)
$$Z_{Pt} = 8.34 \; F \; .15 \, \partial L^* / \partial t,$$

where Z_{Bt} and Z_{Pt} are the appropriate retention time derivatives.

The oxygen demand rate per unit volume is calculated by equation (30)

Because of the greater concentration of active mass of this system, compared to the aeration-only system, the oxygen demand rate is higher for any given retention time.

As explained in the text, installed cost, in thousands of dollars of the complete-mixing, sludge-return activated sludge system can be estimated by:

$$(44) \qquad C_c = a\left[\left(\frac{t}{12.5}\right)F\right]^{\alpha} .$$

In the calculations reported, the "USPHS" values of a = 379 and α = .61 were used. The daily capital cost in dollars per day is found by multiplying C_c by a capital consumption allowance (interest and depreciation).

Daily operating costs were assumed to be proportional to the rate of oxygen transfer per unit volume times the tank's volume. Operating costs were thus proportional to dO/dt (equation (30)) times Ft. A value of $250 per 10^6 gallons was assumed for 12.5 hours retention time. Thus, operating costs per day may be expressed as:

$$(45) \qquad C_o = \frac{k_9L^*(t) + k_2M_a(t)}{k_9L^*(12.5) + k_2M_a(12.5)} \cdot \frac{Ft}{12.5} \cdot 250,$$

where L^* and M_a are calculated by equations (1) and (6).

Total cost is then:

$$(46) \qquad C_A = CKD\ C_c + C_o.$$

The economic optimization equations involve the derivatives of C_A with respect to both t and F. The time derivative is simply:

$$(47) \qquad C_{At} = CKD\,\frac{\alpha C_c}{t} + B^*\left[\left(k_9\frac{\partial L^*}{\partial t} + k_2\frac{\partial M_a}{\partial t}\right) t + k_9L^* + k_2M_a\right],$$

where:

$$B^* = \frac{250F}{12.5(k_9 L^* (12.5) + k_2 M_a (12.5))}.$$

The derivative with respect to F is

$$(48) \qquad C_{AF} = CKD \quad \frac{\alpha C_c}{F} + \frac{C_o}{F}.$$

Equation (48) takes account of the larger tank and rate of oxygen transfer needed to keep retention time and the proportion of wastes removed constant, as flow is increased. The larger flow will carry with it a larger amount of wastes, however, and pollution removal charges will increase if the proportion of waste removed is constant. Assume that there are two pollutants of relevance, phenols and BOD, and write:

$$(49) \qquad \text{Marginal flow cost} = C_{AF} + P_P Z_{PP} W_{PF} + P_B Z_{BB} W_{BF}.$$

The derivatives Z_{ii} are the rate of discharge of pollutants with respect to the rate of inflow of the corresponding waste, holding retention time constant, and can be estimated by the ratio Z_i/W_i. For BOD and phenols:

$$Z_{BB} = Z_B / W_B = L_e / L_i \, ;$$

$$Z_{PP} = Z_P / W_P = L^* / L_i \, .$$

If the concentration of each waste is constant, then:

(50a) $$W_{BF} = 8.34 \, L_i$$

(50b) $$W_{PF} = .15 \cdot 8.34 \, L_i = 1.25 \, L_i \, .$$

To simplify the following discussion, define:

(51) $$Z_{BF} = Z_{BB} \, W_{BF} = 8.34 \, L_e$$

(52) $$Z_{PF} = Z_{PP} \, W_{PF} = 1.25 \, L^*$$

Then, the condition for minimizing the cost of the firm's output is that its level of waste treatment be adjusted so that:

(53) $$C_{At} = -\Sigma_i \, P_i \, Z_{iT} \cdot \qquad P_B \, Z_{Bt} + P_P \, Z_{Pt}$$, where P_B and Pp are respectively the pollution charge per pound of BOD and phenols.

If the firm faces direct regulation rather than explicit pollution charges, then the shadow price equivalent for each pollutant cannot be calculated independently of the price of the other. However, at the extreme, one of the two prices can equal zero, with the entire incentive effect operating through the other price. For $P_P = O$, we have:

Marginal flow cost $= C_{AF} + Z_{BF} \, P_B$

(54) $$= C_{AF} + Z_{BF} \, C_{At} \; / \; Z_{Bt} \; ,$$

where the terms on the right are respectively calculated by equations (48), (51),

(47) and (42).

Similarly, for $P_B = O$:

(55) Marginal flow cost = $C_{AF} + Z_{PF} \; C_{At} \; / \; Z_{Pt}$.

For a given retention time, the true marginal cost of flow in the treatment unit lies between these two magnitudes.

B. Trickling Filters

The basic equations for trickling filters were taken from Galler and Gotaas.[37] The equation relating effluent BOD to influent BOD and design parameters was reported above. Trickling filter costs were written:

$$C_F = N \{ CKD' [(2a+1)(D+1) C_1 \pi / 27$$
$$+ a^2 C_2 \pi + Da^2 C_3 \pi / 27$$
$$+ 2a C_4 + r / (C_6 + C_7 r)]$$
$$+ 4.496 C_5 r (D+4) \}, \tag{56}$$

where N = the number of filters used, D = filter depth in feet, a = filter radius in feet, r = recirculation in millions of gallons per day, CKD' = .20/360., and the C_i's are the cost parameters detailed in Table 2. The terms in the summation are, respectively, the costs of the filter walls, floor, packing, distributor, recirculation pump, and pump operating cost. The authors cited used a mathematical programming technique to derive least cost values of D, a, and r for given influent flow F, raw waste BOD concentration L_i and effluent BOD concentration L_e. The parameters were constrained *a priori* within the following bounds:

(57) $0 \leq r \leq 4F$ (million gal/day)

$3 \leq D \leq 10$ (ft.)[38a]

[37] Two articles, *op. cit.*

[38a] Optimization calculations were also reported using deeper filters.

Table 2: Trickling Filter Cost Parameters and Values Used in Cost Calculations

C_1 = cost of concrete in walls

= $\$80/yd^3$

C_2 = cost of floor

= $\$4/ft^2$

C_3 = cost of packing

= $\$10/yd^3$

C_4 = cost of distribution system

= \$53/ft. diameter

C_5 = power cost

= \$.02/KWH

C_6 = .000555 pump cost factor

C_7 = .01 pump cost factor

Freeboard and wall thickness each assumed to be one foot; pump efficiency, 70%

$$\max\,[10, \sqrt{(I + r)\ \ 462.18}] \le a \le 100 \qquad \text{(ft.)}$$

where I = F/N, the flow through each filter.

For each value of I and L_i used, Galler and Gotaas reported that the "least cost" path for the parameters as L_e was successively reduced involved increasing D to its maximum value, then r, then a. For simplicity, we will refer to these stages as Stage I, II, and III, respectively. The minimum value of a is either 10 ft. or a term corresponding to a maximum hydraulic load (including recirculation) of 30 million gallons per day per acre of filter surface.

The filter's performance is a function of these three design parameters, of the rate of flow of raw wastes, and their concentration. The authors cited used a performance equation of the form:

$$(58) \qquad L_e = \frac{1.067\,(IL_i + r\,L_e)^{1.19}}{(I + r)^{.78}\,(D + 1)^{.67}\,a^{.25}\,I^{.28}} \qquad \text{38b}$$

If influent and effluent BOD concentrations and the rate of flow are known, the values of the parameters may be calculated in each of the three design "stages." In Stage I, r and a are given[39] and D may be calculated by:

$$(59) \qquad D = \frac{1.102\,(I\,L_i + r\,L_e)^{1.78}}{(I + r)^{1.16}\,L_e^{\,1.49}\,a^{.373}\,I^{.417}} - 1.$$

[38b] Assuming an operating temperature of 20° C.

[39] $r = O$ and $a = \max\left[10, \sqrt{(462.18\,(I + r)\,)}\right]$. Even though r = O, using the Galler and Gotaas cost minimizing path, it has been entered explicitly since the derivative of (59) with respect to L_e is used for the alternative cost minimizing path described below.

If this D turned out to be greater than 10 feet, D was set equal to 10 and Stage II calculations were performed.

Equation (58) cannot be solved analytically for r. However, r was approximated using an iterative procedure based on the following version of the basic trickling filter performance equation:

$$(60) \qquad r = \frac{1.087\,(I\,L_i + r\,L_e)^{1.52}}{(D+1)^{.86}\,a^{.32}\,I^{.372}\,L_e^{1.28}} - I$$

where $a = \max\left[10, \sqrt{462.18\,(I + r)}\right]$

For $r > 4I$, Stage III calculations were performed for $r = 4I$, using:

$$(61) \qquad a = \frac{1.196\,(I\,L_i + r\,L_e)^{4.76}}{(D+1)^{2.68}\,I^{1.12}\,L_e^{4}}$$

If the calculated <u>a</u> exceeded 100 ft., the number of filters was increased by one, the waste water flow divided equally among them, and the calculations repeated.

There were three marginal costs reported in the text, denoted, respectively, by MC_D, MC_r, and MC_a. These are of the form,

$$(62) \qquad MC_\alpha = \frac{\partial C}{\partial \alpha}\frac{\partial \alpha}{\partial L_e} \bigg/ \frac{\partial Z_B}{\partial L_e}$$

where α stands for any of D, r, or a. For the minimum cost filter, $MC_D = MC_r = MC_a$, otherwise the same removal performance could be obtained at a lower cost by increasing the low-marginal-cost parameter and decreasing the high-marginal-cost one.

The last derivative on the right in equation (62) is calculated simply:

(63) $$\partial Z_B / \partial L_e = 8.34\ F,$$

since $Z_B = 8.34\ FL_e$.

The other terms are calculated from equations (56) or (58). Turning first to the D derivatives,

(64) $$\frac{\partial C}{\partial D} = N \{ CKD' [(2a + 1) C_1 + C_3 a^2) \pi / 27] + 4.496\ C_5 r \},$$

and, from equation (59),

(65) $$\frac{\partial D}{\partial L_e} = \frac{1.78\ (D+1)r}{I L_i + rL_e} - \frac{1.49(D+1)}{L_e}$$

Skipping to Stage III, the cost gradient with respect to a is:

(66) $$\frac{\partial C}{\partial a} = N \cdot CKD' \{ 2\pi [(C_1 (D + 1) + C_3\ a\ D) / 27 + a\ C_2] + 2\ C_4 \}.$$

The effect on a of performance is, from equation (61):

(67) $$\frac{\partial a}{\partial L_e} = \frac{-4a}{L_e} + \frac{4.76\ a\ r}{(IL_i + rL_e)} .$$

In Stage II, a may also be a function of r, because of the limit on hydraulic loading per filter area. Write equation (58) symbolically as:

$$L_e = f(r,\ L_e,\ a^H(r))$$

where $a^H(r) = \sqrt{(F + r)\ 462.18}$ for $(I + r) \geq .216$ [40]

or $a^H(r) = 10$ for $(I + r) \leq .216$

[40] $(I + r) = 10^2 / 462.18 = .216$ at the critical hydraulic loading for the minimum radius filter of 10 ft.

Then

$$\frac{\partial L_e}{\partial r} = \frac{\frac{\partial f}{\partial r} + \left(\frac{\partial f}{\partial a}\,\frac{\partial a^H}{\partial r}\right)}{1 - \frac{\partial f}{\partial L_e}}$$

and its reciprocal is the desired derivative:

(68) $$\frac{\partial r}{\partial L_e} = \frac{1 - \frac{1.19\, rL_e}{(IL_i + r\, L_e)}}{\frac{1.19 L_e^2}{(IL_i + r\, L_e)} - \frac{.78 L_e}{(I + r)} - \frac{.25 L_e}{a}\,\frac{\partial a^H}{\partial r}}$$

The last term in the denominator, $\partial a^H / \partial r$, is either zero if a = 10 or calculated by:

(69) $$\frac{\partial a}{\partial r} = \frac{231.09}{a}, \text{ for } a > 10.$$

Finally,

(70) $$\frac{\partial C}{\partial r} = N \left\{ CKD' \left[\frac{1}{(C_6 + C_7 r)} - \frac{C_7\, r}{(C_6 + C_7 r)^2} + \frac{\partial C}{\partial a}\,\frac{\partial a^H}{\partial r} \right] + 4.496\, C_5\, (D + 4) \right\},$$

where $\partial C/\partial a$ is calculated by equation (66) and $\partial a^H/\partial r$, as before, may be either zero or positive.

In preliminary calculations, the path for the design parameters suggested by Galler and Gotaas did not equate the three marginal costs, even when the "low cost" parameter was well within its permitted range. In order to approximate more closely a least-cost path, as L_e was successively reduced, the design parameters were successively incremented by small amounts. The criteria for selection at each increment was marginal cost, the parameter with the lowest cost being chosen at each step. The resulting costs, for equivalent removal efficiencies, were several per cent lower than the costs using the Galler and Gotaas path.

The marginal cost of flow was calculated, as for the activated sludge unit, holding the amount of BOD discharged constant. The marginal flow cost, holding L_e constant, takes on the general form:

$$C_{FF\alpha} = \frac{\partial C}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial I} \frac{\partial I}{\partial F} = \frac{\partial C}{\partial \alpha} \frac{\partial \alpha}{\partial I} \cdot \frac{1}{N}$$

where α stands for any of the three design parameters and $\partial C/\partial \alpha$ is calculated as above. The marginal effect of increased flow on D is, from equation (29):

(71) $$\frac{\partial D}{\partial I} = \frac{1.78\ (D+1)\ L_i}{(IL_i + rL_e)} - \frac{1.16(D+1)}{I + r} - \frac{.417\ (D+1)}{I}$$

Differentiating (61),

(72) $$\frac{\partial a}{\partial I} = \frac{4.76\ a\ L_i}{(IL_i + rL_e)} - \frac{1.12a}{I} .$$

Equation (60) may be written symbolically as:

(73) $$r = g\ (I,\ h(I,\ r),\ a^H\ (r,\ I)\ ),$$

where $h(I,\ r) = (IL_i + r\ L_e)^{1.52}$

The derivative is:

(74) $$\frac{\partial r}{\partial I} = \frac{\dfrac{\partial g}{\partial I} + \dfrac{\partial g}{\partial h}\dfrac{\partial h}{\partial I} + \dfrac{\partial g}{\partial a}\dfrac{\partial a^H}{\partial I}}{1 - \dfrac{\partial g}{\partial h}\dfrac{\partial h}{\partial r} - \dfrac{\partial g}{\partial a}\dfrac{\partial a^H}{\partial r}} .$$

The reciprocal of (74) is evaluated by:

$$(75) \qquad \frac{I}{r} = \frac{1 - \dfrac{1.52\,XL_e}{(IL_i + rL_e)} + \dfrac{3.2X}{a}\dfrac{\partial a^H}{\partial r}}{\dfrac{1.52\,XL_i}{(IL_i + rL_e)} - \dfrac{.372X}{I} - 1} \quad + \quad \frac{1.52\,XL_i}{(IL_i + rL_e)} - \frac{3.2X}{a}\frac{\partial a^H}{\partial I}$$

where $X = r + I$ and the two maximum-hydraulic-load derivatives, $\partial a^H/\partial r = \partial a^H/\partial I = O$ for $a = 10$ or $= 231.09/a$ for *a* greater than 10.

As before, this derivative must be corrected to take account of the increased discharges of pollutants which would occur if flow is increased while holding L_e constant. Recall that only BOD removal performance for trickling filters can be predicted from this data. If simplifying assumptions analagous to those made in the activated sludge analysis are also made here, then P_B can be estimated by MC_α at the equivalent treatment level. Corrected marginal flow cost then becomes:

(76) Marginal flow cost $= C_{FF\alpha} + 8.34\ L_e\ MC_\alpha$.

Results of calculations using these formulae will be presented in a forthcoming study of the economics of water use in petroleum refining, by Herbert Mohring and J. Hayden Boyd.